FETI Based Domain Decomposition Methods for Large-scale Topology Optimization

Sivasankar Arul^{1,a)}, Tomáš Brzobohatý,¹ and Tomáš Kozubek¹

¹*IT4Innovations National Supercomputing Center, VSB - Technical University of Ostrava, Ostrava, Czech Republic*

^{a)}Corresponding author: sivasankar.arul@gmail.com

Abstract. This work deals with the issues specific to the development of a massively parallel topology optimization solver. This is developed as a module in the open source software ExaScale PaRallel FETI Solver (ESPRESO, <http://espreso.it4i.cz/>). In a topology optimization problem, the heterogeneity of the system increases, which adversely affects the performance of the Finite Element Tearing and Interconnection (FETI) method. Robust FETI variants which can handle such heterogeneities are discussed here. Their performance is tested and the results are presented. A suitable FETI method is identified for implementation in topology optimization solver.

INTRODUCTION

Topology optimization (TO) is an effective design tool which gives optimum layout of material within the available design space. The density based TO with a MATLAB code for various problems like compliance minimization, compliant mechanism synthesis, etc., is discussed in [1]. In TO, for a static structural linear problem, finite element elasticity problem is solved at every optimization iteration. A scaling parameter named the density, ranging from 0 to 1, is added to each finite element. This parameter is initialized to a uniform value. At every optimization iteration, it changes towards either 0 or 1 indicating whether the element should be present or void. The global stiffness matrix is assembled from the element stiffness matrices. The global matrix undergoes change in coefficients as the individual element stiffness matrix is scaled based on the density from an arbitrary minimum value, say 10^{-3} to its nominal value. This leads to larger variation of coefficients and increasing condition number of the global system matrix, as more and more optimization iterations are carried out.

Large domains with extremely fine finite element meshes lead to large system matrices. Domain Decomposition Methods (DDMs) are used for computation of solution for such large systems. There are various DDMs such as multigrid method [2], balancing domain decomposition [3] or Finite Element Tearing and Interconnection (FETI)[4]. FETI method has good numerical scalability and robustness, provided the substructures resulting from decomposition are homogeneous, has smooth boundaries and has good aspect ratios. In TO, there are heterogeneities due to jumps in coefficient (stiffness) across substructures, resulting in slow convergence or breakdown of the FETI method. The behaviour of FETI-DP in a TO solver is discussed in [5]. However, the domain is assumed to have one material. So the Young modulus is scaled and set as 1. For a general purpose TO solver, the Young modulus cannot be scaled. Robust variations of FETI methods, which have good convergence in the presence of heterogeneities and bad aspect ratio, have been proposed in [6, 7, 8]. As a baseline work, these FETI variants have been implemented and tested for suitability for a TO problem.

FETI VARIANTS

Variants of the Total-FETI (TFETI) [9, 10] are compared in this section. The first variant is TFETI using projected conjugate gradient method with Dirichlet preconditioner (PCGP-FETI). The second variant is PCGP-FETI with full orthogonalization of search directions (FO-FETI).

In many cases, the poor convergence is due to the bad approximation of characteristics of global problem using local information of the subdomains. By solving eigenvalue problem on the interface of each subdomain, called Generalized eigenvalue in the overlap (GenEO) [6], a second coarse space using problematic modes is generated. Deflation of the interface problem using this coarse space restores the convergence of FETI method for heterogeneous problems. It is important to mention that there is significant overhead while building the second coarse space due to the solutions of eigenvalue problems.

Simultaneous FETI (S- FETI) [7] uses the additive structure of the preconditioner to extend the standard conjugate gradient algorithm to a multi-preconditioned conjugate gradient. This involves generating several search directions that originate from the preconditioners and solving the interface problem iteratively.

Block-FETI [7] algorithm is a Block CG method [11]. The FETI linear system is rewritten in block form. The solution is updated at the same time as the Krylov subspaces are computed. Several isolated eigenvalues are captured within one iteration.

Performance of FETI variants

FIGURE 1. (a) Chessboard type domain with alternating materials. (b-d) Decomposition of the domain into 8, 16, and 32 subdomains respectively. The subdomain interface is not aligned with the material distribution.

FIGURE 2. Convergence behaviour for (a) PCGP-FETI and (b) FO-FETI from 2 to 32 subdomains.

A chessboard type domain is used to test the performance of the FETI variants. The domain consists of 9 sub blocks. There are two materials Mat01 and Mat02 and ratio of the Young moduli is 10^5 . Alternate sub block is assigned a particular material as shown in Fig. 1(a). METIS [12] is used for decomposition of the domain into subdomains. Figure 1(b-d) shows the decomposition by METIS into 8, 16, and 32 subdomains. The decomposition is not aligned

FIGURE 3. Convergence behaviour for (a) GenEO-FETI, (b) S-FETI, and (c) Block-FETI from 2 to 32 subdomains.

FIGURE 4. Convergence behaviour for FETI variants for (a) 2 and (b) 32 subdomains.

with material distribution and hence there can be jumps in the stiffness along the interface of these subdomains. The domain is fixed on the left part of the boundary and a force is applied on the right part of the boundary. With increasing number of subdomains, number of iterations rapidly increases for the standard FETI approach (PCGP-FETI) as shown in Fig. 2(a). Full orthogonalization of the search directions (FO-FETI) reduces the number of iterations as shown in Fig. 2(b), but it is still large. Hence, numerical scalability of FETI method vanishes. Figure 3 shows the convergence behaviour for GenEO-FETI, S-FETI, and Block-FETI respectively from 2 to 32 subdomains. GenEO-FETI method significantly reduces the number of iterations. S-FETI and Block-FETI have the best convergence behaviour and they take almost the same number of iterations for convergence even as the number of subdomains increases. A comparison of convergence behaviour of all variants for 2 and 32 subdomains is shown in Fig. 4. S-FETI and Block-FETI show good numerical scalability and are suitable for TO problems. Although S-FETI has good convergence behaviour, it requires larger computational effort than the standard FETI method. [13, 14] propose an adaptive strategy which reduces the computational effort without losing robustness. [15] proposes alternative implementations of S-FETI, reducing memory usage and preserving time performance. It should be noted that in this work only numerical scalability is evaluated. The parallel scalability of these methods has to be investigated as well. FETI method will have good convergence if the domain decomposition is done with respect to material properties. However, it is impossible to do this type of decomposition in optimization algorithm, since the jumps are randomly scattered in geometry according to optimization process [5]. It should also be noted that a TO problem will have greater heterogeneity since the element can be void or present randomly in any portion of the domain.

CONCLUSION

This work was aimed to develop a massively parallel TO solver within ESPRESO library utilizing FETI domain decomposition method. Different variants of FETI have been implemented and their performance has been tested on a chessboard type problem having material jumps. S-FETI and Block-FETI show good numerical scalability. They will be tested for a TO problem and will be suitable for the solver provided they have good parallel scalability.

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