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Magnetooptické jevy v nanostrukturách se
sníženou symetrií

Magneto-optical effects in nanostructures
with reduced symmetry

Author: Tibor Fördös

Supervisor: doc.Dr.Mgr.Kamil Postava

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Student: **Tibor Fördös**

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Magneto-optical effects in nanostructures with reduced symmetry

Description:

Cílem bakalářské práce je teoretický popis magnetooptických jevů v nanostrukturách a jejich využití pro studium magnetických vlastností systémů se sníženou symetrií. Hlavními cíly jsou zejména:

- popis magnetooptických jevů v multivrstvách na základě šíření elektromagnetických vln v gyrotropním prostředí
- popis lineárních kvadratických magnetooptických jevů v reflexi od materiálu se sníženou symetrií
- experimentální studium systémů krystalických feromagnetických částic Fe, Co a FeCo, interpretace a modelování výsledků

Main goals of the bachelor thesis are theoretical description of magneto-optical effects in nanostructures and their applications in magnetic study in systems with reduced symmetry. The targets are:

- description of magneto-optical effects in multilayers based on electromagnetic wave propagation in gyrotropic
- description of linear and quadratic magneto-optical effects in reflection from materials with reduced symmetry
- experimental investigation of systems of Co, Fe, and FeCo crystalline ferromagnetic particles, discussion and modeling of results.

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- P. Yeh, Optical waves in layered media, John Wiley & Sons, 1988.
- B. E. A. Saleh, M. C. Teich, Fundamentals of photonics, 2nd Ed., John Wiley & Sons, 2007.

Extent and terms of a thesis are specified in directions for its elaboration that are opened to the public on the web sites of the faculty.

Supervisor: **doc. Dr.Mgr. Kamil Postava**

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prof. Ing. Petr Noskievič, CSc.
Vice-rector for Study Affairs

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Abstrakt

Teoretická část bakalářské práce je zaměřena na teoretický popis magnetooptických jevů ve strukturách se sníženou symetrií. Užitím Yehového maticového formalismu je odvozen přibližný vztah pro výpočet komplexního magnetooptického úhlu od polonekonečného ortorombického krystalu. Citlivost kvadratických jevů na symetrii krystalu může být využita ke studiu symetrií ve strukturách. Experimentální část je zaměřena na charakterizaci vrstev, které se skládají z feromagnetických Fe a Co nanočástic v prostředí SiO_2 . Magnetické vlastnosti byly měřeny s využitím longitudinálního magnetooptického Kerrova jevu. Z měření magnetooptických hysterezních smyček vyplývá rozdíl mezi Fe nanočásticovou vrstvou se silnou in-plane jednoosou anizotropií a téměř izotropní Co vrstvou. Tloušťky vrstev a objemový podíl částic byly stanoveny s využitím spektroskopické elipsometrie. Elipsometrická data byla fitována modelem efektivního prostředí.

Abstract

Theoretical part of this bachelor thesis is focused to the theoretical description of magneto-optical effects in reflection from structures with reduced symmetry. Using Yeh's matrix formalism, the appropriate formulae for the magneto-optical effect from semi-infinite orthorombic crystal is derived. The sensitivity of quadratic magneto-optical effects to crystal symmetry can be used for study of symmetries of the structures. Experimental part is focused on characterization of layers consisting of the ferromagnetic Co and Fe nanoparticles embedded in SiO_2 matrix. Magnetic properties were measured using longitudinal magneto-optical Kerr effect. Magneto-optical hysteresis loop measurements show differences between Fe particular film with strong in-plane uniaxial anisotropy and Co one, which is almost isotropic. Nanoparticulate film thicknesses and volume fractions of particles were obtained using spectroscopic ellipsometry. Ellipsometric data were fitted to the model based on the Bruggeman effective medium approximation.

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Nomenclature

$A^{(n)}$	wave amplitude in the n -th layer
$\mathbf{A}$	rotation matrix
$\mathbf{B}$	magnetic flux density
$\mathbf{D}$	electric displacement
$\mathbf{D}^{(n)}$	dynamic matrix of the n -th layer
$\mathbf{E}$	electric field intensity
G_{ijkl}	components of the linear magneto-optical tensor
$\mathbf{H}$	magnetic field intensity
$\mathbf{J}$	Jones matrix
K_{ijk}	components of the linear magneto-optical tensor
$\mathbf{M}$	magnetization vector
$\mathbf{M}$	total matrix
N_x, N_y, N_z	effective refractive index components
$\mathbf{P}^{(n)}$	propagation matrix of the n -th layer
$d^{(n)}$	thickness of the n -th layer
$\mathbf{e}^{(n)}, \mathbf{h}^{(n)}$	normalized eigen-polarization in the n -th layer
$\mathbf{j}$	current density
k	wave number
r_{ij}	reflection coefficient
t	time
$\hat{x}, \hat{y}, \hat{z}$	unit vectors
$\Gamma, \Gamma_{\pm}^*$	parameters of the anisotropy
$\hat{\epsilon}^{(n)}$	permittivity tensor of the n -th layer
$\epsilon_{s,p}, \theta_{s,p}$	ellipsometric angles
$\phi_{s,p}$	complex magneto-optical angle
ω	angular frequency of monochromatic wave
ρ	volume density of the free charges

Chapter 1

Introduction

Magneto-optics (MO) deals with the electromagnetic response of the system during presence or change of magnetization. Magneto-optical effects are widely applied to study of magnetism of structures - from bulk to nanostructures. Technique based on these effects, for example magneto-optical ellipsometry, are very sensitive nondestructive methods suitable for study of magnetic anisotropy, magnetization reversal properties of nanostructures, and exchange coupling[1,12]. Practical applications of magneto-optical effects are e.g. in constructions of magnetic field sensors [3] and magnetophotonic optical insulators [4].

It is usual, that the MO models of the structures are obtained using numerical methods. On the other hand, analytical investigation of electromagnetic response of the magnetized structure gives us physical insight into these effects and good understanding of their symmetry properties. Although the analytical formulas are even in the simple situations very extensive and complicate, some approximations allow to obtain formulas in compact and synoptical forms. One of them is ultrathin film approximation. In this thesis, others of them are employed. **The approximation of small magneto-optical angles** is very appropriate because of the fact, that these angles are in most cases $\sim \text{mrad}$. This fact also allows us to use the 2-order Taylor polynomial approximation, which keeps investigated linear and quadratic terms. The higher-order effects are also neglected. **The near normal incidence approximation** ($\varphi^{(0)} \ll 1 \rightarrow \sin \varphi^{(0)} \approx \varphi^{(0)}$) allows to study effects, which appear in the case of small angle of incidence $\varphi^{(0)}$ in the contrast to the normal incidence. This configuration is also experimentally more applicable. These approaches were used for investigation of magneto-optical response in reflection from crystals with diverse symmetries e.g. a cubic crystal magnetized parallel to the interface, an orthorombic crystal magnetized in special directions of magnetization vector at

the normal incidence [5], a uniaxial crystal with a general magnetization direction at the near normal incidence [6] etc. There is a need to understand dependence of linear and quadratic magneto-optical effects for general magnetization direction in orthorombic crystal.

Main object of this thesis is analytical description of magneto-optical effects in structures with reduced symmetry. As will be shown, some of these effects are very sensitive on the crystal symmetry. This fact could be used for study of symmetries of the structures. First, it is necessary to comprehend mathematical method for solving the Maxwell equations in anisotropic system. One of them is the Yeh's matrix formalism, which is described in Chapter 2. The origin of the anisotropy is discussed in the Chapter 3. In our case, the anisotropy induced with the magnetic field coexist with the anisotropy, which is caused by reduced symmetry of crystals. Chapter 4 deals with analytical analysis of magneto-optical effects in orthorombic and uniaxial crystals. MO effects in such structures are usually calculated numerically, but theoretical formulae is important for understanding of the dependencies of these effects on the crystals symmetries.

In the Chapter 5, we describe characterization of layers consisting of the ferromagnetic Co and Fe single crystal particles embedded in SiO_2 matrix. Magnetic properties of the particular layers are studied using magneto-optical ellipsometry and optical properties of the layers are measured using spectroscopic ellipsometry. Ferromagnetic metal particles in dielectric matrix possess interesting magnetic and optical properties [7,8].

Chapter 2

Electromagnetic response of anisotropic multi-layers

In this chapter, the Yeh's matrix formalism is described[9]. This formalism provides effective mathematical method for description of optical properties of anisotropic multilayered system.

2.1 Wave equation for the anisotropic media

The Maxwell equations are partial differential equations, which describe classical properties of the electromagnetic field. They can be written in the following differential form

$$\text{div } \mathbf{D} = \rho \quad (2.1)$$

$$\text{div } \mathbf{B} = 0 \quad (2.2)$$

$$\text{rot } \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad (2.3)$$

$$\text{rot } \mathbf{H} = \mathbf{j} + \frac{\partial \mathbf{D}}{\partial t}, \quad (2.4)$$

where $\mathbf{E}$, $\mathbf{H}$, $\mathbf{D}$, $\mathbf{B}$, ρ , and $\mathbf{j}$ denote the electric field intensity, the magnetic field intensity, the electric displacement, the magnetic flux density, the volume density of the free charges, and the current density, respectively. One of the solutions is a monochromatic plane wave, which is described by following equations

$$\mathbf{E}(\mathbf{r}, t) = \Re\{\mathbf{E}_0 \exp[i(\omega t - \mathbf{k}\mathbf{r})]\} \quad (2.5)$$

$$\mathbf{H}(\mathbf{r}, t) = \Re\{\mathbf{H}_0 \exp[i(\omega t - \mathbf{k}\mathbf{r})]\}, \quad (2.6)$$

where $\mathbf{k}$ and ω are the wave vector and the angular frequency. The time course of the direction of $\mathbf{E}(r, t)$ determine **polarization state** of the light, which is commonly described using Jones vector - a column matrix in the form

$$\mathbf{J} = \begin{bmatrix} E_{x0} \\ E_{y0} \end{bmatrix},$$

where E_{x0} and E_{y0} are the x and y components of the complex amplitude $\mathbf{E}_0$ in the case of a wave propagating in the z-direction[10,11]. It is common to define polarization state relative to the plane of incidence of light on the surface. We use s- and p- polarizations (see Fig 4.1). Then, we can define the reflections coefficient

Figure 2.1: S-polarization lies along x-axis - perpendicular to the plane of incidence. P-polarization lies in the plane of incidence.

$$r_{ij} = \frac{E_{0j}}{E_{0i}}, \quad (2.7)$$

where the indices denote i (incident s- or p-) and j (reflected s- or p-).

By substituting (2.5) and (2.6) into last two Maxwell equations (2.3) and (2.4), we obtain the condition for the complex amplitudes $\mathbf{E}_0$ and $\mathbf{H}_0$

$$\mathbf{k} \times \mathbf{E}_0 - \omega \mu_0 \hat{\mu} \mathbf{H}_0 = 0 \quad (2.8)$$

$$\mathbf{k} \times \mathbf{H}_0 + \omega \varepsilon_0 \hat{\varepsilon} \mathbf{E}_0 = 0,$$

where ε_0 and μ_0 denotes the vacuum permittivity and permeability. Note, that for optical frequencies the permeability tensor $\hat{\mu} \approx 1$ can be employed. After eliminating $\mathbf{H}$, we obtain the wave equation in the form

$$k_0 \hat{\varepsilon} \mathbf{E}_0 - \mathbf{k}^2 \mathbf{E}_0 + \mathbf{k}(\mathbf{k} \mathbf{E}_0) = 0, \quad (2.9)$$

where the permittivity tensor $\hat{\varepsilon}$ describe anisotropy of the environment and can be generally written in the form

$$\hat{\varepsilon} = \begin{pmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{pmatrix}.$$

Factual forms of $\hat{\varepsilon}$ will be discussed in Chapter 3.

2.2 Matrix formalism

Let us consider the N -multilayer system with the paralel interfaces normal to the z axis. Each layer n is characterized by a tensor $\hat{\varepsilon}^{(n)}$ and the thickness $d^{(n)}$.

Then, the wave equation (2.9) could be for each layer rewritten as[12,13]

$$k_0 \hat{\varepsilon}^{(n)} \mathbf{E}_0^{(n)} - \mathbf{k}^2 \mathbf{E}_0^{(n)} + \mathbf{k}^{(n)} (\mathbf{k}^{(n)} \mathbf{E}_0^{(n)}) = 0, \quad (2.10)$$

where the wave vector $k^{(n)} = k_0(N_x \hat{\mathbf{x}} + N_y \hat{\mathbf{y}} + N_z \hat{\mathbf{z}})$ obey the Snell's law - invariance of the components N_x and N_y in each layer. Let us have $N_x = 0$ and $N_y = n^{(0)} \sin \varphi^{(0)} = \text{const.}$, which could be fulfilled by choosing appropriate coordinate system. Now, the wave equation (2.10) can be rewritten in the elegant matrix form

$$\underbrace{\begin{pmatrix} \varepsilon_{xx}^{(n)} - N_y^2 - N_z^{(n)2} & \varepsilon_{xy}^{(n)} & \varepsilon_{xz}^{(n)} \\ \varepsilon_{yx}^{(n)} & \varepsilon_{yy}^{(n)} - N_z^{(n)2} & \varepsilon_{yz}^{(n)} + N_y N_z^{(n)} \\ \varepsilon_{zx}^{(n)} & \varepsilon_{zy}^{(n)} + N_y N_z^{(n)} & \varepsilon_{zz}^{(n)} - N_y^2 \end{pmatrix}}_{\heartsuit} \begin{bmatrix} E_{0x}^{(n)} \\ E_{0y}^{(n)} \\ E_{0z}^{(n)} \end{bmatrix} = 0, \quad (2.11)$$

where the condition for existence of nontrivial solution is ensured by

$$\det(\heartsuit) = 0, \quad (2.12)$$

which is generally a quartic equation. Also we obtain four solution $N_{zj}^{(n)}$, where $(j = 1, 3)$ and $(j = 2, 4)$ correspond to forward and backward propagating modes, and the

corresponding proper polarizations $\mathbf{E}_{0j}^{(n)}$, which can be rewritten in the form

$$\mathbf{E}_{0j}^{(n)} = A_j^{(n)} \mathbf{e}_{0j}^{(n)}, \quad (2.13)$$

where $\mathbf{e}_{0j}^{(n)}$ is the normalized eigen-polarization.

For the expression of the field vector $\mathbf{E}^{(n)}$ is employed so-called Fraunhofer approximation. The idea of this approximation is based on the possibility of a superposition of corresponding polarizations in each place of the structure. Thus, inside n-th layer at the interface n/n+1 we have

$${}^{(n/n+1)}\mathbf{E}_0^{(n)} = \sum_{j=1}^4 A_j^{(n)} \mathbf{e}_{0j}^{(n)}. \quad (2.14)$$

The propagation in the n-th layer change the field vector according to a factor $\exp[ik_0 N_{zj}^{(n)} d^{(n)}]$

$${}^{(n-1/n)}\mathbf{E}_0^{(n)} = \sum_{j=1}^4 A_j^{(n)} \mathbf{e}_{0j}^{(n)} \exp[ik_0 N_{zj}^{(n)} d^{(n)}]. \quad (2.15)$$

The boundary conditions require continuity of tangential components of the field vectors $\mathbf{E}$ and $\mathbf{H}$ at the interface. Therefore

$$\begin{aligned} {}^{(n-1/n)}\mathbf{E}_{0x}^{(n)} &= {}^{(n-1/n)}\mathbf{E}_{0x}^{(n-1)}, \quad {}^{(n-1/n)}\mathbf{E}_{0y}^{(n)} = {}^{(n-1/n)}\mathbf{E}_{0y}^{(n-1)}, \\ {}^{(n-1/n)}\mathbf{H}_{0x}^{(n)} &= {}^{(n-1/n)}\mathbf{H}_{0x}^{(n-1)}, \quad {}^{(n-1/n)}\mathbf{H}_{0y}^{(n)} = {}^{(n-1/n)}\mathbf{H}_{0y}^{(n-1)}. \end{aligned}$$

These requirements can be rewritten in the compact matrix form

$$\mathbf{D}^{(n-1)} \mathbf{A}^{(n-1)} = \mathbf{D}^{(n)} \mathbf{P}^{(n)} \mathbf{A}^{(n)},$$

where $\mathbf{D}^{(n)}$ (Dynamic matrix) and $\mathbf{P}^{(n)}$ (Propagation matrix) have the form

$$\mathbf{D}^{(n)} = \begin{pmatrix} \mathbf{e}_{x1}^{(n)} & \mathbf{e}_{x2}^{(n)} & \mathbf{e}_{x3}^{(n)} & \mathbf{e}_{x4}^{(n)} \\ \mathbf{h}_{y1}^{(n)} & \mathbf{h}_{y2}^{(n)} & \mathbf{h}_{y3}^{(n)} & \mathbf{h}_{y4}^{(n)} \\ \mathbf{e}_{y1}^{(n)} & \mathbf{e}_{y2}^{(n)} & \mathbf{e}_{y3}^{(n)} & \mathbf{e}_{y4}^{(n)} \\ \mathbf{h}_{x1}^{(n)} & \mathbf{h}_{x2}^{(n)} & \mathbf{h}_{x3}^{(n)} & \mathbf{h}_{x4}^{(n)} \end{pmatrix}, \quad \mathbf{A}^{(n)} = \begin{bmatrix} A_1^{(n)} \\ A_2^{(n)} \\ A_3^{(n)} \\ A_4^{(n)} \end{bmatrix} \quad (2.16)$$

$$\mathbf{P}^{(n)} = \begin{pmatrix} ik_0 N_{z1}^{(n)} d^{(n)} & 0 & 0 & 0 \\ 0 & ik_0 N_{z2}^{(n)} d^{(n)} & 0 & 0 \\ 0 & 0 & ik_0 N_{z3}^{(n)} d^{(n)} & 0 \\ 0 & 0 & 0 & ik_0 N_{z4}^{(n)} d^{(n)} \end{pmatrix}. \quad (2.17)$$

Now, we are able to describe the relation between the amplitude in the halfspaces (0) and $(N+1)$. We obtain

$$\mathbf{A}^{(0)} = \underbrace{[\mathbf{D}^{(0)}]^{-1} \mathbf{D}^{(1)} \mathbf{P}^{(1)} \dots [\mathbf{D}^{(N)}]^{-1} \mathbf{D}^{(N+1)} \mathbf{P}^{(N+1)}}_{\mathbf{M}} \mathbf{A}^{(N+1)}, \quad (2.18)$$

where $\mathbf{M}$ is so-called total matrix of the system.

Let us consider no backward propagating waves in halfspace $(N + 1)$. Thus $A_2^{N+1} = A_4^{N+1} = 0$. Then the reflection coefficients for incident s-polarized wave ($A_3^{(0)} = 0$) can be written as

$$r_{12} = r_{ss} = \frac{A_2^{(0)}}{A_1^{(0)}} = \frac{M_{21}M_{33} - M_{23}M_{31}}{M_{11}M_{33} - M_{13}M_{31}} \quad (2.19)$$

$$r_{14} = r_{sp} = \frac{A_4^{(0)}}{A_1^{(0)}} = \frac{M_{41}M_{33} - M_{43}M_{31}}{M_{11}M_{33} - M_{13}M_{31}} \quad (2.20)$$

and simmiliary for incident p-polarization we obtain ($(A_1^{(0)} = 0)$)

$$r_{32} = -r_{ps} = \frac{A_2^{(0)}}{A_3^{(0)}} = \frac{M_{11}M_{23} - M_{13}M_{21}}{M_{11}M_{33} - M_{13}M_{31}} \quad (2.21)$$

$$r_{34} = -r_{pp} = \frac{A_4^{(0)}}{A_3^{(0)}} = \frac{M_{11}M_{43} - M_{13}M_{41}}{M_{11}M_{33} - M_{13}M_{31}}. \quad (2.22)$$

In our case, note, that indices 1,2,3, and 4 denote eigen-polarizations - incident s, reflected s, incident p, and reflected p, respectively.

Chapter 3

Magneto-optical material tensors

This chapter deals with the permittivity tensor in the media with reduced symmetry. In this case, the anisotropy induced with the magnetic field coexist with the anisotropy, which is caused by reduced symmetry of crystals.

3.1 Mathematical definition of Cartesian tensors

Tensors provide a sophisticated mathematical framework for describing and solving many physical problems. Many physical quantities cannot be described by using scalars or vectors. An example is the permittivity ε in anisotropic media, where physical properties are not the same in all directions. In those cases one can use tensor analysis. Tensor is the n -rank matrix, which can be measured in any Cartesian frame and which has a physical significance *independent* of the frame. In general, a tensor of rank n is a mathematical object with n suffixes, $T_{ijk\dots}$, which obeys the transformation law

$$T_{ijk\dots} = a_{ip}a_{jq}a_{kr\dots}T_{pqr\dots} \quad (3.1)$$

where a are the components of the rotation matrix $\mathbf{A}$ between frames. The product (3.1) represent the Cartesian tensor transformation. The Einstein summation convention over coordinates x, y, z is employed.

3.2 Magneto-optical permittivity tensor

The components of the permittivity tensor $\hat{\varepsilon}$ can be expanded into the series[14]

$$\varepsilon_{ij} = \sum_{w=0}^{\infty} \varepsilon_{ij}^{(w)} = \varepsilon_{ij}^{(0)} + \varepsilon_{ij}^{(1)} + \varepsilon_{ij}^{(2)} + \dots = \varepsilon_{ij}^{(0)} + K_{ijk}M_k + G_{ijkl}M_kM_l + \dots \quad (3.2)$$

where $\varepsilon_{ij}^{(0)}$ are the components of the permittivity tensor without the magnetization ($\mathbf{M} = 0$) and K_{ijk} and G_{ijkl} are the components of the linear and quadratic magneto-optical tensors.

Because of the Onsager's principle[15]

$$\varepsilon_{ij}(\mathbf{M}) = \varepsilon_{ji}(-\mathbf{M}) \quad (3.3)$$

or the symmetries

$$\begin{aligned} \varepsilon_{ij}^{(1)} &= -\varepsilon_{ji}^{(1)} \\ \varepsilon_{ij}^{(2)} &= \varepsilon_{ji}^{(2)}, \end{aligned} \quad (3.4)$$

the tensor elements can be written as

$$\begin{aligned} K_{iik} &= 0 \\ K_{ijk} &= -K_{jik} \\ G_{ijkl} &= G_{jikl} = G_{ijlk} = G_{jilk}. \end{aligned} \quad (3.5)$$

Therefore, we can write following general equations for the permittivity components in the reduced form

$$\begin{pmatrix} \varepsilon_{yz}^{(1)} \\ \varepsilon_{zx}^{(1)} \\ \varepsilon_{xy}^{(1)} \end{pmatrix} = \begin{pmatrix} K_{yzx} & K_{yzy} & K_{yzz} \\ K_{zxx} & K_{zxy} & K_{zzx} \\ K_{xyx} & K_{xyy} & K_{xyz} \end{pmatrix} \begin{pmatrix} M_x \\ M_y \\ M_z \end{pmatrix} \quad (3.6)$$

$$\begin{pmatrix} \varepsilon_{xx}^{(2)} \\ \varepsilon_{yy}^{(2)} \\ \varepsilon_{zz}^{(2)} \\ \varepsilon_{yz}^{(2)} \\ \varepsilon_{zx}^{(2)} \\ \varepsilon_{xy}^{(2)} \end{pmatrix} = \begin{pmatrix} G_{xxxx} & G_{xxyy} & G_{xxzz} & G_{xxyz} & G_{xxzx} & G_{xxxy} \\ G_{yyxx} & G_{yyyy} & G_{yyzz} & G_{yyyz} & G_{yyzx} & G_{yyxy} \\ G_{zzxx} & G_{zzyy} & G_{zzzz} & G_{zzyz} & G_{zzzx} & G_{zzxy} \\ G_{yzxx} & G_{yzyy} & G_{yzzz} & G_{yzyz} & G_{yzzx} & G_{yzxy} \\ G_{zxxx} & G_{zxyy} & G_{zxzz} & G_{zxyz} & G_{zxzx} & G_{zxxy} \\ G_{xyxx} & G_{xyyy} & G_{xyzz} & G_{xyyz} & G_{xyzx} & G_{xyxy} \end{pmatrix} \begin{pmatrix} M_x^2 \\ M_y^2 \\ M_z^2 \\ M_yM_z \\ M_zM_x \\ M_xM_y \end{pmatrix}. \quad (3.7)$$

The general tensors in the above equations are describing the triclinic structure.

3.2.1 Magneto-optical tensors in crystals with reduced symmetry

In the following discussion, we consider two important crystal symmetries - uniaxial (trigonal, tetragonal, and hexagonal) and orthorhombic. The derivation of the particular forms of the tensors is based on the symmetry operations. These operations doesn't change the tensor form. The rotations about the axes x, y and z are described by the matrixes.

$$\begin{aligned}
 A_x(\theta) &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta \\ 0 & \sin \theta & \cos \theta \end{pmatrix} \\
 A_y(\varphi) &= \begin{pmatrix} \cos \varphi & 0 & \sin \varphi \\ 0 & 1 & 0 \\ -\sin \varphi & 0 & \cos \varphi \end{pmatrix} \\
 A_z(\phi) &= \begin{pmatrix} \cos \phi & -\sin \phi & 0 \\ \sin \phi & \cos \phi & 0 \\ 0 & 0 & 1 \end{pmatrix}
 \end{aligned} \tag{3.8}$$

Let's have the principal axes of the orthorhombic crystal parallel to the Cartesian axes. Then, the non-magnetic permittivity tensor $\varepsilon^{(0)}$ has the form

$$\begin{pmatrix} \varepsilon_{xx} & 0 & 0 \\ 0 & \varepsilon_{yy} & 0 \\ 0 & 0 & \varepsilon_{zz} \end{pmatrix} \tag{3.9}$$

This tensor describe the uniaxial structure with the optical axis parallel to z axis in the limit case $\varepsilon_{xx}^{(0)} = \varepsilon_{yy}^{(0)}$.

Linear magneto-optical tensors in uniaxial crystals

The general tensor in (3.6) is transformed according to (3.1), where the transformation matrix from (3.8) $A_z = C_{nz}$ represents the symmetry operation of the rotation about z-axis by the angle $\phi = \pm \frac{2\pi}{3}$ (trigonal structure - C_3), $\phi = \pm \frac{\pi}{2}$ (tetragonal structure - C_4) and $\phi = \pm \frac{\pi}{3}$ (hexagonal structure - C_6). From the equality $K'_{ijk} = K_{ijk}$ we obtain the linear tensor in the form

$$\begin{pmatrix} K_{yzx} & -K_{yzy} & 0 \\ K_{yzy} & K_{yzx} & 0 \\ 0 & 0 & K_{xyz} \end{pmatrix}. \tag{3.10}$$

When we take into account the symmetry operation of the rotation about x axis by the angle $\theta = \pi$, which can be represented by the matrix

$$C_{2x} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \quad (3.11)$$

we obtain the linear tensor in the form

$$\begin{pmatrix} K_{yzx} & 0 & 0 \\ 0 & K_{yzx} & 0 \\ 0 & 0 & K_{xyz} \end{pmatrix}, \quad (3.12)$$

which is the same for all three uniaxial structures.

Linear magneto-optical tensor in orthorombic crystals

Let us have the orthorombic axes parallel to the Cartesian axes. By using the symmetry operation the rotation by the angle π about the orthorombic axes x, y and z, which can be described by the following matrixes

$$C_{2x} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \quad (3.13)$$

$$C_{2y} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \quad (3.14)$$

and

$$C_{2z} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (3.15)$$

we obtain the linear tensor of the orthorombic system in the following form

$$\begin{pmatrix} K_{yzx} & 0 & 0 \\ 0 & K_{zxy} & 0 \\ 0 & 0 & K_{xyz} \end{pmatrix}. \quad (3.16)$$

Quadratic magneto-optical tensors

One can use similar approach to derive the quadratic magneto-optical tensors. Tensors for all 32 crystal classes were derived in [5]. Some of them are in the table 3.1. The asterisks indicate the additional terms for the structure symmetry missing the operation of a two-fold in-plane axis (C_{2x} or C_{2y}).

Table 3.1: Quadratic magneto-optical tensors

Trigonal	$\begin{pmatrix} G_{xxxx} & G_{xxyy} & G_{xxzz} & 2G_{xxyz} & 2G_{xxxz}^* & 2G_{xxxy}^* \\ G_{xxyy} & G_{xxxx} & G_{xxzz} & -2G_{xxyz} & -2G_{xxxz}^* & -2G_{xxxy}^* \\ G_{zzxx} & G_{zzxx} & G_{zzzz} & 0 & 0 & 0 \\ G_{yzxx} & -G_{yzxx} & 0 & 2G_{yzyz} & 2G_{yzxz}^* & -2G_{zxxx}^* \\ G_{zxxx}^* & -G_{zxxx}^* & 0 & -2G_{yzxz}^* & 2G_{yzyz} & 2G_{yzxx} \\ -G_{xxxy}^* & G_{xxxy}^* & 0 & -2G_{xxxz}^* & 2G_{xxyz} & G_{xxxx} - G_{xxyy} \end{pmatrix} \quad (3.17)$
Tetragonal	$\begin{pmatrix} G_{xxxx} & G_{xxyy} & G_{xxzz} & 0 & 0 & 2G_{xxxy}^* \\ G_{xxyy} & G_{xxxx} & G_{xxzz} & 0 & 0 & -2G_{xxxy}^* \\ G_{zzxx} & G_{zzxx} & G_{zzzz} & 0 & 0 & 0 \\ 0 & 0 & 0 & 2G_{yzyz} & 2G_{yzxz}^* & 0 \\ 0 & 0 & 0 & -2G_{yzxz}^* & 2G_{yzyz} & 0 \\ G_{xyxx}^* & -G_{xyxx}^* & 0 & 0 & 0 & 2G_{xyxy} \end{pmatrix} \quad (3.18)$
hexagonal	$\begin{pmatrix} G_{xxxx} & G_{xxyy} & G_{xxzz} & 0 & 0 & 2G_{xxxy}^* \\ G_{xxyy} & G_{xxxx} & G_{xxzz} & 0 & 0 & -2G_{xxxy}^* \\ G_{zzxx} & G_{zzxx} & G_{zzzz} & 0 & 0 & 0 \\ 0 & 0 & 0 & 2G_{yzyz} & 2G_{yzxz}^* & 0 \\ 0 & 0 & 0 & -2G_{yzxz}^* & 2G_{yzyz} & 0 \\ -G_{xxxy}^* & G_{xxxy}^* & 0 & 0 & 0 & G_{xxxx} - G_{xxyy} \end{pmatrix} \quad (3.19)$
Orthorombic	$\begin{pmatrix} G_{xxxx} & G_{xxyy} & G_{xxzz} & 0 & 0 & 0 \\ G_{xxyy} & G_{xxxx} & G_{xxzz} & 0 & 0 & 0 \\ G_{zzxx} & G_{zzxx} & G_{zzzz} & 0 & 0 & 0 \\ 0 & 0 & 0 & 2G_{yzyz} & 0 & 0 \\ 0 & 0 & 0 & 0 & 2G_{zxxx} & 0 \\ 0 & 0 & 0 & 0 & 0 & 2G_{xyxy} \end{pmatrix} \quad (3.20)$

Chapter 4

MO effects in crystals with reduced symmetry

In this chapter, analytical analysis of magneto-optical effect will be realized. We show the dependencies of these effects on the crystals symmetries.

4.1 Analytical formulae

For description of the magneto-optical response we use the complex magneto-optical angle

$$\phi_{s,p} = \theta_{s,p} + i\epsilon_{s,p}, \quad (4.1)$$

where two magneto-optical angles, Kerr rotation $\theta_{s,p}$ and ellipticity $\epsilon_{s,p}$, can be defined for incident s - and p -polarized light using approximation of small angles

$$\frac{r_{sp}}{r_{ss}} = \frac{\tan \theta_s + i \tan \epsilon_s}{1 - i \tan \theta_s \tan \epsilon_s} \approx \theta_s + i \epsilon_s \quad (4.2)$$

$$-\frac{r_{ps}}{r_{pp}} = \frac{\tan \theta_p + i \tan \epsilon_p}{1 - i \tan \theta_p \tan \epsilon_p} \approx \theta_p + i \epsilon_p. \quad (4.3)$$

The sign definition is demonstrated in Fig. 4.1. Let us consider the semi-infinite orthorhombic crystal, which has axes parallel to the Cartesian axes. Then, total matrix of the system has the form

$$\mathbf{M} = [\mathbf{D}^{(0)}]^{-1} \mathbf{D}^{(1)}, \quad (4.4)$$

where $\mathbf{D}^{(0)}$ and $\mathbf{D}^{(1)}$ denote the dynamic matrix of the incident medium and the dynamic matrix of the crystal, respectively. Let us modify the tensors defined in Chapter 3 in the

Figure 4.1: Definition of the positive MO angles for s- and p- incident linear polarizations.

following form

$$\hat{\epsilon} = \begin{pmatrix} \epsilon_{xx} + \epsilon_{xx}^{(2)} x^2 & \epsilon_{xy}^{(1)} x + \epsilon_{xy}^{(2)} x^2 & -\epsilon_{zx}^{(1)} x + \epsilon_{zx}^{(2)} x^2 \\ -\epsilon_{xy}^{(1)} x + \epsilon_{xy}^{(2)} x^2 & \epsilon_{yy} + \epsilon_{yy}^{(2)} x^2 & \epsilon_{yz}^{(1)} x + \epsilon_{yz}^{(2)} x^2 \\ \epsilon_{zx}^{(1)} x + \epsilon_{zx}^{(2)} x^2 & -\epsilon_{yz}^{(1)} x + \epsilon_{yz}^{(2)} x^2 & \epsilon_{zz} + \epsilon_{zz}^{(2)} x^2 \end{pmatrix}. \quad (4.5)$$

The near normal incidence approximation, where $N_y = n^{(0)} \sin \varphi^{(0)} \approx n^{(0)} \varphi^{(0)}$, was employed. Using the Yeh's matrix formalism, the formulae for the MO complex angle was derived. At final, the 2-order Maclaurin polynomial approximation, which keeps linear (x) and quadratic terms (x^2), was used. We obtain the following formulae

$$\begin{aligned} \phi_{s,p} &= \frac{2\epsilon_{xy}^{(1)} n^{(0)}}{(n^{(0)} \mp \sqrt{\epsilon_{xx}})(n^{(0)} \pm \sqrt{\epsilon_{yy}})(\sqrt{\epsilon_{xx}} + \sqrt{\epsilon_{yy}})} \pm \frac{2n^{(0)}(\epsilon_{zx}^{(1)} \epsilon_{yz}^{(1)} - \epsilon_{xy}^{(2)} \epsilon_{zz})}{\epsilon_{zz}(n^{(0)} \mp \sqrt{\epsilon_{xx}})(n^{(0)} \pm \sqrt{\epsilon_{yy}})(\sqrt{\epsilon_{xx}} + \sqrt{\epsilon_{yy}})} \\ &\mp \varphi^{(0)} \frac{2n^{(0)2} \sqrt{\epsilon_{yy}}(\epsilon_{zx}^{(1)} \pm \epsilon_{zx}^{(2)})}{\epsilon_{zz}(n^{(0)} \mp \sqrt{\epsilon_{xx}})(n^{(0)} \pm \sqrt{\epsilon_{yy}})(\sqrt{\epsilon_{xx}} + \sqrt{\epsilon_{yy}})} \pm \varphi^{(0)} \frac{2n^{(0)2} \epsilon_{xy}^{(1)} \epsilon_{yz}^{(1)}}{\epsilon_{zz}(n^{(0)} \mp \sqrt{\epsilon_{xx}})(n^{(0)} \pm \sqrt{\epsilon_{yy}})^2(\sqrt{\epsilon_{xx}} + \sqrt{\epsilon_{yy}})} \end{aligned} \quad (4.6)$$

In the case of the normal incidence, the reflection coefficients have the following compact form

$$r_{12} = r_{ss} = \frac{n_0 - \sqrt{\epsilon_{xx}}}{n_0 + \sqrt{\epsilon_{xx}}} - \frac{n_0 \epsilon_{xy}^{(1)2} (n_0 + 2\sqrt{\epsilon_{xx}} + \sqrt{\epsilon_{yy}})}{\sqrt{\epsilon_{xx}}(n_0 + \sqrt{\epsilon_{xx}})^2 (n_0 + \sqrt{\epsilon_{yy}})(\sqrt{\epsilon_{xx}} + \sqrt{\epsilon_{yy}})^2} \quad (4.7)$$

$$r_{14} = r_{sp} = \frac{2\epsilon_{xy}^{(1)} n_0}{(n_0 + \sqrt{\epsilon_{xx}})(n_0 + \sqrt{\epsilon_{yy}})(\sqrt{\epsilon_{xx}} + \sqrt{\epsilon_{yy}})} + \frac{2n_0(\epsilon_{zx}^{(1)} \epsilon_{yz}^{(1)} - \epsilon_{xy}^{(2)} \epsilon_{zz})}{\epsilon_{zz}(n_0 + \sqrt{\epsilon_{xx}})(n_0 + \sqrt{\epsilon_{yy}})(\sqrt{\epsilon_{xx}} + \sqrt{\epsilon_{yy}})} \quad (4.8)$$

$$r_{32} = -r_{ps} = -\frac{2\epsilon_{xy}^{(1)} n_0}{(n_0 + \sqrt{\epsilon_{xx}})(n_0 + \sqrt{\epsilon_{yy}})(\sqrt{\epsilon_{xx}} + \sqrt{\epsilon_{yy}})} + \frac{2n_0(\epsilon_{zx}^{(1)} \epsilon_{yz}^{(1)} + \epsilon_{xy}^{(2)} \epsilon_{zz})}{(n_0 + \sqrt{\epsilon_{xx}})(n_0 + \sqrt{\epsilon_{yy}})(\sqrt{\epsilon_{xx}} + \sqrt{\epsilon_{yy}})} \quad (4.9)$$

$$r_{34} = -r_{pp} = \frac{n_0 - \sqrt{\epsilon_{yy}}}{n_0 + \sqrt{\epsilon_{yy}}} - \frac{n_0 \epsilon_{xy}^{(1)2} (n_0 + 2\sqrt{\epsilon_{yy}} + \sqrt{\epsilon_{xx}})}{\sqrt{\epsilon_{yy}}(n_0 + \sqrt{\epsilon_{xx}})(n_0 + \sqrt{\epsilon_{yy}})^2 (\sqrt{\epsilon_{xx}} + \sqrt{\epsilon_{yy}})^2} \\ - \frac{n_0 \sqrt{\epsilon_{yy}}(\epsilon_{yz}^{(1)2} + \epsilon_{yz}^{(2)} \epsilon_{zz})(n_0 + 2\sqrt{\epsilon_{yy}} n_0 + 2\frac{\epsilon_{xx}}{\epsilon_{yy}} n_0 + \frac{3}{2}\frac{\epsilon_{xx}}{\epsilon_{yy}} + 2\frac{\epsilon_{xx}}{\sqrt{\epsilon_{yy}}} + \sqrt{\epsilon_{xx}})}{\epsilon_{zz}(n_0 + \sqrt{\epsilon_{xx}})(n_0 + \sqrt{\epsilon_{yy}})^2 (\sqrt{\epsilon_{xx}} + \sqrt{\epsilon_{yy}})^2}. \quad (4.10)$$

As can be seen, in the case without magnetization, expressions (4.7) and (4.10) turn to the expressions known as the Fresnel equations. Expressions derived for orthorhombic system describe uniaxial structure in the special case $\epsilon_{22} = \epsilon_{11}$, indeed.

In the following sections, the MO complex angle (expression (4.6)) will be discussed. Note, that three components of the magnetization vector are defined - polar, longitudinal and transverse (see Fig. 4.2).

Figure 4.2: Basic MO configurations - polar, longitudinal and transverse component of magnetization vector[1].

4.2 Linear magneto-optical effects

The first fraction in (4.6) depends on $\varepsilon_{xy}^{(1)}$, which is equal to $\varepsilon_{xy}^{(1)} = K_{xyz}M_z$ for all uniaxial and orthorombic crystals. This term describes the linear polar effect. Thus, components M_x and M_y do not influence the linear MO response in the case of normal incidence $\varphi^{(0)}=0$.

However, in the case of near normal incidence, contribution $\varepsilon_{zx}^{(1)}$ appears (see the third term). In the case of orthorombic crystal $\varepsilon_{zx}^{(1)} = K_{zxy}M_y$ and uniaxial crystal $\varepsilon_{zx}^{(1)} = K_{yzx}M_y$ determine the linear longitudinal effect, respectively, and do not depend on M_x and M_z . However, in the case of uniaxial crystal missing in-plane two-fold symmetry axis (* indicates additional term), we obtain $\varepsilon_{zx}^{(1)} = K_{yzy}^*M_x + K_{yzx}M_y$. Thus, contribution of M_x and M_y are mixed together and can be separated.

I should note, that the linear polar effect can be used for sensing of polar component of magnetization vector M_z at the normal incidence. On the other hand, the oblique incidence configuration could be useful for determination of the in-plane magnetization components M_x and M_y .

4.3 Quadratic magneto-optical effects

Orthorombic crystal

In the case of the normal incidence, we can observe quadratic effects, which depend on the in-plane magnetization components. First, in the second term we have a product of $\varepsilon_{zx}^{(1)} = K_{zxy}M_y$ and $\varepsilon_{yz}^{(1)} = K_{yzx}M_x$. Second contribution is given by quadratic tensor element $\varepsilon_{xy}^{(2)} = 2G_{xyxy}M_xM_y$.

When we setup near normal incidence, new quadratic contributions appear. In the third term we have $\varepsilon_{zx}^{(2)} = 2G_{zxzx}M_zM_x$ and in the fourth term appears a product of $\varepsilon_{xy}^{(1)} = K_{xyz}M_z$ and $\varepsilon_{yz}^{(1)} = K_{yzx}M_x$. Those new contributions also do not depend on the longitudinal component M_y . We will not consider the rotation of the crystal about z axis, because the tensor element $\varepsilon_{xy}^{(0)}$ is not included in our calculation.

Uniaxial crystal

In the contrast to the orthorombic structure, permittivity components $\varepsilon_{ij}^{(0)}$ and $\varepsilon_{ij}^{(1)}$ of the uniaxial crystal are independent on the crystal rotation about z axis. Thus, complex MO effect depends only on the quadratic components $\varepsilon_{xy}^{(2)}$ and $\varepsilon_{zx}^{(2)}$, which are specific for each uniaxial structure. This fact could be very useful for investigation of the symmetries of the structures.

Let us consider **tetragonal** structure including and missing* two-fold in-plane symmetry axis. We obtain

$$\varepsilon_{xy}^{(2)} = 2G_{xyxy}M_xM_y - G_{xyxx}^*(M_y^2 - M_x^2). \quad (4.11)$$

Let us rotate with the crystal by angle ϕ about z axis. According to (3.1) and (3.8) we obtain the tensor component $\varepsilon_{xy}^{(2)}$ in the case of symmetry including two-fold in-plane axis in the form

$$\varepsilon_{xy}^{(2)} = [\Gamma - \Gamma\cos(4\phi) + 2G_{xyxy}]M_xM_y - \frac{1}{2}\Gamma\sin(4\phi)(M_y^2 - M_x^2), \quad (4.12)$$

where the parameter $\Gamma = \frac{1}{2}(G_{xxxx} - G_{xyxy} - 2G_{xyxy})$. In the case of symmetry missing two-fold in-plane axis we obtain

$$\begin{aligned} \varepsilon_{xy}^{(2)} = & \left[\Gamma - \Gamma\cos(4\phi) + 2G_{xyxy} + \sin(4\phi)\Gamma_+^* \right] M_xM_y \\ & - \left[\frac{1}{2}\Gamma\sin(4\phi) + \frac{1}{2}(\Gamma_-^* - \Gamma_+^*\cos(4\phi)) \right] (M_y^2 - M_x^2), \end{aligned} \quad (4.13)$$

where specific parameter $\Gamma_{\pm}^* = G_{xxxy} \pm G_{xyxx}$ allows us to distinguish between our structures. Note, that the component $\varepsilon_{zx}^{(2)}$ does not depend on the sample rotation about the z axis.

The case of **hexagonal** symmetry is simple, because the quadratic tensor is independent on the rotation about z axis. For the tensor component $\varepsilon_{xy}^{(2)}$ we obtain

$$\varepsilon_{xy}^{(2)} = (G_{xxxx} - G_{xxyy})M_x M_y + G_{xxxy}^*(M_y^2 - M_x^2). \quad (4.14)$$

As can be seen, the term $G_{xxxy}^*(M_y^2 - M_x^2)$ allows us to distinguish between our structures.

For **trigonal** symmetry we obtain

$$\begin{aligned} \varepsilon_{xy}^{(2)} = & (G_{xxxx} - G_{xxyy})M_x M_y + 2(G_{xxyz}\sin 3\phi - G_{xxzx}^*\cos 3\phi)M_y M_z \\ & + 2(G_{xxyz}\cos 3\phi + G_{xxzx}^*\sin 3\phi)M_z M_x + G_{xxxy}^*(M_y^2 - M_x^2) \end{aligned} \quad (4.15)$$

For in-plane magnetization is the formula same as in the case of hexagonal symmetry. The courses of periodical functions is evidently specific for each structure. Moreover, the component $\varepsilon_{zx}^{(2)}$ depends on the sample rotation

$$\begin{aligned} \varepsilon_{zx}^{(2)} = & 2[G_{yzxx}\cos 3\phi + G_{zxxx}^*\sin 3\phi]M_x M_y - 2G_{yzxz}^*M_y M_z \\ & + 2G_{yzyz}M_z M_x + [G_{yzxx}\sin 3\phi - G_{zxxx}^*\cos 3\phi](M_y^2 - M_x^2). \end{aligned} \quad (4.16)$$

Chapter 5

Experimental part

In this chapter, we describe characterization of layers consisting of the ferromagnetic Co and Fe single crystal particles embedded in SiO_2 matrix using magneto-optical and optical ellipsometry [16]. All samples were prepared at Academy of sciences of the Czech Republic.

5.1 Samples structures

Nanoparticulate Co and Fe films were prepared by using dual radiofrequency (RF) magnetron sputtering with alternating targets. Figure 5.1 shows structure of the samples. First, the SiO_2 film of thickness of 10 nm was deposited on the crystal Si substrate at room temperature. Particles of ferromagnetic metal were formed on the top of the SiO_2 film. The system of particles was covered by SiO_2 , which filled interparticle volume, capped the nanoparticulate layer, and smoothed the top surface. We have investigated systems with one ($N = 1$) and ten ($N = 10$) nanoparticulate layers (see Fig. 5.1). Nominal effective thickness of the ferromagnetic nanoparticulate layers are 3, 5, and 10 nm. The growth rates were 2.22 nm/min, 3.76 nm/min, and 1.48 nm/min for Fe, Co, and SiO_2 , respectively. The HRTEM observations were made by using JEOL JEM 3010 (300 kV, LaB_6 cathode and 1.7 Å point resolution) with an energy dispersive X-ray detector. As one can see in Figure 5.2, Fe particles are joined to each other, while Co particles are mostly completely surrounded by host SiO_2 .

Figure 5.1: Left subplot shows schematically the layer structure of the samples. Observations of Fe multilayer structure – 10 nm of SiO₂ (light layers) sandwiched with 10 nm (left TEM image) and 3 nm (right TEM image) layers of Fe nanoparticles embedded in SiO₂ matrix.

Figure 5.2: Observations of 3 nm layers of Fe nanoparticles (in the above pictures) and Co nanoparticles (in the below pictures) embedded in SiO₂ matrix.

5.2 Magneto-optical ellipsometry

Magneto-optical ellipsometry is a very sensitive measurement technique based on a polarization change of light reflecting by magnetic sample [1]. We can describe magneto-optical response of a sample by using magneto-optical angles, which are defined for incident s - and p -polarized light using approximation of small angles (4.3). In this paper we use the s -polarized incident light, thus the reflected light is polarized elliptically with the ellipticity ϵ_s and the azimuth rotation θ_s .

Contribution of the three component of the magnetization vector, which defined in the previous chapter, can be mixed together. Their separation can be made by using 180° sample and magnetic field rotation. This idea is based on fact, that the polar contribution is symmetric, while longitudinal is antisymmetric [17]. Thus, separation can be done by addition and subtraction of measured data.

Our experimental set-up consists of semiconductor laser working at wavelength of $\lambda = 670$ nm, polarizer, quarter-wave plate as a retarder, Wollaston prism, and two photodiodes (see 5.3). The method is so called differential intensity method. We have measured longitudinal and transversal hysteresis loops for incident s -polarized light as a function of the external in-plane magnetic field H . The angle of incidence was set to $\varphi = 60^\circ$ and the maximum external magnetic field of $H_{max} = 1$ kOe was applied.

Figure 5.3: MO experimental setup

5.3 Optical ellipsometry

Ellipsometric characterization is based on the change of the incident light polarization by a sample. Classical ellipsometry enables to measure the amplitude and the phase of

the complex reflection coefficients ratio

$$\frac{r_{pp}}{r_{ss}} = \tan \psi e^{i\Delta} \quad (5.1)$$

where r_{pp} and r_{ss} are the Fresnel reflection coefficients for p - and s -polarized light and ψ , Δ are the ellipsometric angles.

We use the ellipsometer UVISSEL produced by Jobin-Yvon Horiba company, which is based on phase modulation technology and covers the spectral range from 190 nm to 2100 nm. Our ellipsometric data were measured at the angle of incidence of 70° and were fitted to the model based on the Maxwell-Garnett and Bruggeman effective medium approximation describing nanoparticulate films.

5.4 Results

Longitudinal and transverse hysteresis loops of single layer ($N=1$) of Co and Fe particles were measured for different sample azimuth angles. Figure 5.4 shows the hysteresis loops for easy and hard axis, and the polar plot of remanence effect. Clear two-fold in-plane uniaxial anisotropy confirms that coherent rotation of the magnetization dominates.

In contrast to the Fe sample, the Co nanoparticulate film is almost magnetically isotropic, which is evident from the polar remanence plot shown in Fig. 5.5. Also the shape of the longitudinal hysteresis loop is almost independent on the in-plane magnetic field direction. The origin of this magnetic isotropy is associated with the random distribution of the magnetic properties of nanoparticles, which is also confirmed by rounded shape of the longitudinal hysteresis loop (see Fig. 5.5).

We have studied multilayer samples ($N = 10$) using spectroscopic ellipsometry to increase sensitivity to optical properties of ferromagnetic particulate layers. Complex refractive indexes of the Co and Fe particles embedded in SiO_2 matrix were modeled using the Maxwell-Garnett and Bruggeman effective medium approximations. The Maxwell-Garnett approach describes a system of very diluted collections of small particles completely surrounded by host material [18]. Let us consider the mixture where spherical inclusions, much smaller than the optical wavelength, with permittivity ϵ_i occupy random positions in the environment of permittivity ϵ_e . Then the Maxwell-Garnett mixing formula can be written as

$$\frac{\epsilon_{eff} - \epsilon_e}{\epsilon_{eff} + 2\epsilon_e} = f \frac{\epsilon_i - \epsilon_e}{\epsilon_i + 2\epsilon_e} \quad (5.2)$$

Figure 5.4: Hysteresis loops for magnetic field along in-plane easy ($\phi = 120^\circ$) and hard axis ($\phi = 30^\circ$) of Fe 3 nm thick monolayer in longitudinal and transversal directions. Right figure shows corresponding polar plot.

Figure 5.5: In-plane hysteresis loops ($\phi = 30^\circ$) of 3 nm thick Co monolayer in longitudinal and transversal directions. Right figure shows corresponding polar plot.

where ϵ_{eff} is the effective permittivity and f denotes the volume fraction of the inclusions in the mixtures. The Bruggeman approach describes a composite of aggregated phases or random-mixture structures. The essence is the absolute equality between the phases in the mixture. The Bruggeman formula can be written as

$$(1 - f) \frac{\epsilon_e - \epsilon_{eff}}{\epsilon_e + 2\epsilon_{eff}} + f \frac{\epsilon_i - \epsilon_{eff}}{\epsilon_i + 2\epsilon_{eff}} = 0 \quad (5.3)$$

We have applied a simple model for multilayer sample ($N = 10$) consisting of the 3 nm thick Fe films by using basic optical constants of Fe and SiO₂ [19,20]. As can be seen in Figure 5.6, the Bruggeman model is more appropriate. The reason is associated with the fact, that most of the nanoparticles are joined to each other. The obtained thicknesses are: $t_{SiO_2} = 11.5$ nm, $t_{Fe} = 7.3$ nm and the volume fraction of Fe is $f_{Fe} = 0.36$.

Figure 5.6: Ellipsometric angles ψ and Δ of 3 nm thick Fe multilayers. Comparison between the Maxwell-Garnett model and the Bruggeman model shows that the later fits the data better.

Chapter 6

Conclusion

This thesis enabled me an understading of the Yeh's matrix formalism and the tensor description of the magnetized medium with reduced symmetry with employing the symmetry arguments.

Main contribution of this bachelor thesis are:

- derivation of the complex magneto-optical effects in reflection from orthorombic crystal in the case of near normal incidence(4.6) and the reflection coefficients (4.7)-(4.10)
- study of ferromagnetic nanoparticular films using magneto-optical ellipsometry - magnetic anisotropy was studied using linear longitudinal magneto-optical Kerr effect
- study of ferromagnetic nanoparticular films using optical ellipsometry - the thicknesses and the volume fractions of the samples were obtained using the Bruggeman effective medium approximations

Experimental results were presented at the Joint European Magnetic Symposia 2010 (JEMS) and will be published in the Journal of Physics: Conference Series [16].

Main goals for future work

- application of the theoretical description of the quadratic magneto-optical effects to the experiment
- study the magneto-optical effects in the crystals with employing the microscopic models
- origin of the anisotropy of Fe and Co nanoparticular samples

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