Financial Applications of the Conditional Expectation

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1 Introducción

In this thesis, we study some financial applications of the conditional expectation and apply this work to three distinguished problems in finance. The main component that connects these three issues is the notion of conditioning, an important concept in probability and statistics, which turn out to be extremely useful in financial modeling. The conditional expectation $E(Y|X)$, represents the best estimate of the random variable $Y$ given the available information about $X$. Given the importance that conditional expectation plays in modern finance and in several pricing and risk management problems.

First, we extensively use the conditional expectation to provide some theoretical motivations behind the use of the moving average rule as one of the most popular trading tools among practitioners. In particular, we examine the conditional probability of the price increments and we study how this probability changes over time. Then, we compare the ex-post wealth obtained using these trading rules and other portfolio strategies. The ex-post analysis confirms that it is better using these rules to predict the market trends. In this context, we suggest a methodology that incorporates moving average rules as alarm rules to predict potential fails of the market.

Second, we present different approaches to evaluate the presence of the arbitrage opportunities in the option market. In this context, we propose alternative approaches to estimate the state price density using the conditional expectation estimators. In particular, we use two different methodologies to evaluate the conditional expectation of a random variable $X$ given a random variable $Y$, namely the kernel method and the OLP method recently proposed by Ortobelli et al. (2015). The kernel nonparametric regression method allows estimating the regression function, which is a realization of the conditional expectation $E(Y|X)$, while the second approach estimates the conditional expectation (intended as a random variable), based on an appropriate approximation of the $\sigma$-algebra generated by $X$.

Third, we examine the use of the conditional expectation, either to reduce the dimensionality of large-scale portfolio problem or to propose alternative risk-reward performance measures. In particular, we focus on three different financial uses. In the first use, we discuss and examine some correlation measures (based on the conditional expectation) used to approximate properly the returns in large-scale portfolio problems.
Then, we compare the impact of alternative return approximation methodologies on the ex-post wealth of a classic portfolio strategy. In this context, we show that correlation measures that use properly the conditional expectation perform better than the classical ones. Moreover, the correlation measure typically used for returns in the domain of attraction of a stable law works better than the classical Pearson correlation does. In the second usage, we propose new performance measures based on the conditional expectation that takes into account the heavy tails of the return distributions. Then, we examine portfolio strategies based on the optimization of the proposed performance measures. In particular, we compare the ex-post wealth obtained applying portfolio strategies, which use alternative performance measures based on the conditional expectation. Finally, we deal with the portfolio selection problem from the point of view of different non-satiable investors: namely, risk-averse and risk-seeking. Doing so, we propose alternative use of the conditional expectation in different portfolio problems.
2 Objective and Structure

Conditional expectation is an important concept in probability and statistics which turn out to be extremely useful in financial modeling. It plays a crucial role in portfolio theory and in several pricing and risk management problems. The aim of this dissertation is to assess the impact of the conditional expectation on different financial applications, e.g. arbitrage opportunities, state price density estimation and large-scale portfolio selection problems etc. Given uncertainty in the input model and parameters, the goal of the study often becomes the estimation of a conditional expectation among different financial variables. The conditional expectation is expected performance conditioned on the selected model and parameters. The distribution of this conditional expectation describes precisely, and concisely, the impact of input uncertainty on performance prediction. Conceptually, from probability theory perspective, the conditional expectation is well studied and its properties are mainly proved.

Given the importance of technical analysis, we extensively use the conditional expectation to provide theoretical foundations for the most popular rule among practitioners, the moving average rule. This contribution attempts to overcome a significance gap in literature which is that no adequate theoretical support for such strategies exists. Moreover, to contribute to the literature on option pricing theory, we present different approaches to evaluate the presence of the arbitrage opportunities in the option market. Then, we propose alternative methods to estimate the SPD. To achieve this aim, we estimate the density of a conditional expectation using two different approaches, namely the classical kernel estimator and a new method recently proposed by Ortobelli et al. (2015). Finally, the last aim of the thesis is to examine and investigate the implications for portfolio theory of using conditional expectation estimators.

The rest of the dissertation is organized as follows. Chapter 2, contains detailed discussion of the conditional expectation and summaries the financial theory needed for the development of the thesis. Chapter 3, provides theoretical and practical motivation behind the use of moving average rules. Chapter 4, presents some methods to evaluate the arbitrage opportunities and proposes alternative methods to estimate the SPD. Chapter 5, examines and discusses the impact of conditional expectation estimators in the portfolio theory. Finally, chapter 6 concludes the thesis.
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The thesis consists of three main chapters (number 3, 4 and 5), after the introductory chapters. In the third chapter, we discuss and evaluate the use and the impact of the very popular (among practitioners and academics) moving average rule. In particular, we provide some theoretical motivations for the use of the moving average as one of the most popular technical trading rules. In contrast to the vast studies that use the moving average as an indicator function that indicates merely an up or down state of the market, this study sets the theoretical foundation by demonstrating its validity from a statistical point of view under some particular hypothesis. Thus, we prove that when the moving average rule applies, we could have some implications on the up and down trend probabilities. For this reason, we believe that this rule can be better used to predict the probability of market fails, such as during periods of systemic risk as suggested by Tichý et al. (2015) and Giacometti et al. (2015).

The second contribution of this chapter is to implement some popular moving averages rules using daily data of S&P 500 components. Thus, we introduce an alarm rule that predicts the presence of market systemic risk. The alarm is a simple rule that counts the assets whose average returns on the last \( n \) days is lower than the mean on the last \( N \) trading days (\( n < N \)). If the number of these assets reach a benchmark we deduce that systemic risk is probably present on the market and thus we should not invest in any asset. Finally, we evaluate the usefulness of the moving average rules by comparing their ex-post wealth with those obtained from other portfolio strategies.

In the fourth chapter, we present different approaches to evaluate the presence of the arbitrage opportunities in the option market. In particular, we focus on the violation of the put-call parity no-arbitrage relation and then the nonnegativity of the SPD. Firstly, we measure the violation of put-call parity as the difference in implied volatility between call and put options that have the same strike price, the same expiration date and the same underlying asset. Secondly, we discuss the violation of the nonnegativity of the SPD. This is important, because negative values of the SPD immediately correspond to the possibility of free-lunch in the market.

The second crucial contribution is to propose different approaches to estimate the SPD. We deviate from previous studies in that we estimate SPD directly from the underlying asset under the hypothesis of the BS model. To this end we follow two distinguished approaches to recover the SPD, the first one based on nonparametric
estimation techniques “kernel” which are natural candidates (see among others Ait-Sahalia and Duarte, 2003; Benko et al. 2007, for an application to options), then a new method based on conditional expectation estimator proposed by Ortobelli et al. (2015). Firstly, we examine the so called real mean return function using local polynomial smoothing technique. Then, we estimate the conditional expectation under real probability density. According to the hypothesis of BS model, we are able to derive a closed formula for approximating the conditional expectation under risk neutral probability. The main goal of this contribution is to examine and compare the conditional expectation method and the nonparametric technique. These methods allow us extrapolating arbitrage opportunities and relevant information from different markets (futures and options) consistently with the analysis of the underlying

Finally, in the fifth chapter, we investigate the implications for portfolio theory using conditional expectation estimators. In particular, we focus on three financial applications: i) approximation of the conditional expectation within large-scale portfolio selection problems, ii) performance valuation considering the heavy tails of returns, and iii) the optimal portfolio choices for different investors’ preferences.

The first contribution of this chapter is to investigate the impact of alternative return approximation methods depending by $k$-factors in large-scale portfolio problem (such as in the $k$-fund separation model of Ross (1979)). In particular, we examine and compare the classical return approximation with a nonparametric approximation of the returns depending on few factors obtained by a principal components analysis (PCA). Furthermore, according to Ortobelli and Tichý (2015), we determine the principal components (of PCA) either using a correlation matrix suitable for heavy tailed distribution (called stable linear correlation), or using the classical Pearson correlation matrix (which summarizes the joint dispersion behavior of Gaussian vectors). The most commonly used approach to estimate the relationship between returns and $k$ factors is the linear approximation based on the ordinary least squares (OLS) estimator (see Ross (1979)). This approximation appears good enough when the returns are normally distributed. Admitting small departures from normality of the returns do not affect the regression coefficients greatly, however errors with a heavier tailed distribution, which is more suitable for modeling asset returns, can significantly affect the estimated OLS regression coefficients, (see Nolan et al. (2013)). Moreover, we believe that there exists substantial evidence of nonlinearity in the financial dataset used to estimate the returns (see among others Rachev et al 2008). For this reason, according to Ruppert and Wand
we propose a nonparametric regression analysis to approximate the returns. This approach relaxes the assumptions of linearity and it is suitable even for non-Gaussian distributions. In this context, we prove that the variability of errors of the return approximation decreases as the number of factor increases even when elliptically distributed returns present heavy tails. In addition, using concave dominance testing, we find that the nonparametric regression outperforms much better than its counterpart parametric (OLS) does. This empirical analysis is provided using portfolios of the components of S&P 500 index.

The second contribution of this chapter is to deal with a proper evaluation of portfolio choices that account the distributional tails of portfolios. In particular, the main purpose of this contribution is to present theoretically sound portfolio performance measures considering a more realistic behavior of the returns (i.e. heavy tailed distributions). Using a recent alternative conditional expectation estimator proposed by Ortobelli et al. (2015), we are able to forecast the conditional expected portfolio returns with respect to a given sigma algebra of events (either generated by possible profits or generated by possible losses). More specifically, the first suggested performance measure is based on the conditional expectation with respect to two different σ-algebras (the σ-algebra generated by the portfolio losses, and the σ-algebra generated by the portfolio profits). While the second performance measure considers σ-algebras generated by the joint losses and by joint gains in the market. Moreover, we illustrate how the new performance measures can mitigate the shortcoming of the classical Sharpe ratio (see Sharpe (1994)) showing with an ex-post empirical analysis their tested higher capacity to produce wealth in the US market.

The third contribution of this chapter proposes a new consistent multivariate kernel estimator to approximate the conditional expectation, and we stochastically compare the errors of the return approximation. We show that the approximation error is reduced with the new estimator, and that we can optimize the bandwidth parameters (whose approximation is always considered a problem for kernel-type estimators). Then, we deal with the portfolio selection problem from the point of view of different non-satiable investors, namely risk-seeking and risk-averse investors (see Ortobelli et al. (2015)). In particular, by using the conditional expected value properly, we first identify different definitions of returns based on the investors’ preferences. The new definitions of returns are based on the conditional expected value between the random wealth assessed at different times. Finally, we compare the ex-post wealth obtained by maximizing some
well-known performance ratios applied to the different returns definitions. In doing this, we can examine the impacts of the choices of investors with different risk aversion attitudes.

5 Methods Applied

The thesis presents financial applications of the conditional expectation. In the third chapter, we extensively use the conditional expectation to provide some theoretical motivations behind the use of the moving average rule as one of the most popular trading tools among practitioners. In particular, we examine the conditional probability of the price increments and we study how this probability changes over time. Then, we compare the ex-post wealth obtained using these trading rules and other portfolio strategies. In particular, we compare the ex-post wealth obtained maximizing a stochastic timing performance proposed by Ortobelli et al. 2016, and the wealth obtained maximizing the Sharpe ratio when in both cases we use a moving average rule as alarm of systemic risk.

In the fourth chapter, we extensively use the conditional expectation estimators to evaluate the presence of arbitrage opportunities in the option market. In this context, we propose alternative approaches to estimate the state price density using the conditional expectation estimators. In particular, we use two different methodologies to evaluate the conditional expectation of a random variable $X$ given a random variable $Y$, namely the kernel method and the OLP method recently proposed by Ortobelli et al. (2015). The kernel nonparametric regression method allows estimating the regression function, which is a realization of the conditional expectation $E(Y|X)$, while the second approach estimates the conditional expectation (intended as a random variable), based on an appropriate approximation of the $\sigma$-algebra generated by $X$.

In the fifth chapter, we assess the impact of nonparametric techniques based on the use of conditional expectation estimators in the portfolio theory. In particular, we discuss the use of the conditional expectation for three financial applications: a) approximation problems within large-scale portfolio selection problems, b) performance valuation considering the heavy tails of returns and, c) optimal portfolio choices consistent with different investor preferences.
6 Summary of Results and Conclusion

We already stressed in the introductory chapters the importance of the topic: option market, technical analysis and portfolio theory. The aim of this work is to propose theoretical and methodological approaches to cover different portfolio managers’ goals. This thesis contributes to such rich and challenging environment in at least four ways.

In chapter 2 we briefly introduce some of the most important concepts from the probability theory and financial mathematics that are useful in the financial applications of the conditional expectation: we hope that it could represent a useful map for researchers navigating through this vast and growing corpus of resources.

In chapter 3, we find that under some assumptions the probability of up-trend is greater than the probability of down trend. For this reason, we propose to use moving average rules to predict periods of systemic risk. In this context, we suggest a methodology that incorporate moving average rules as alarm rules to predict potential fails of the market. Thus, we examine the impact of the moving average rules on the U.S. stock market. Firstly, a comparison among different moving average trading rules with and without alarms of losses is performed. Secondly, we compare the ex post wealth obtained with the best performing systemic risk rule used as trading strategy with the wealth obtained maximizing two different portfolio performances. From the comparison among different strategies and stochastic dominance tests, we deduce that the best use of the moving average rules is obtained to predict periods of market distress. These empirical analyses suggest that the moving average rules are much more effective and performing when used to detect the presence of systemic risk.

Further research could involve theoretical and empirical studies. On the one hand, investors may employ complex versions of the moving average rules. On the other hand, the impact of calendar periods such as the weekend effect, the turn-of-the-month effect, the holiday effect and the January effect. Future research will investigate this aspects. Another promising direction for future research is to consider other technical indicators, which may be easier to detect algorithmically, to examine whether or not such indicators are able to predict the presence of systemic risk.

In chapter 4, we present alternative approaches to evaluate the presence of the arbitrage opportunities in the option market. In particular, we empirically investigate the well-known put-call parity no-arbitrage relation and the SPD. First, we measure the violation of the put-call parity as the difference in implied volatilities between call and
put options that have the same strike price, the same expiration date and the same underlying asset. Then, we discuss the usefulness of the nonnegativity of the SPD. We evaluate the effectiveness of the proposed approaches by an empirical analysis on S&P 500 index options data. Moreover, we propose alternative approaches to estimate the SPD under the classical hypothesis of the BS model. To this end, we first examine the real mean return function using local polynomial smoothing technique. Then, we estimate the conditional expectation under real probability density. Under the hypothesis of BS model, we are able to derive a closed formula for approximating the conditional expectation under risk neutral probability.

We use the classical nonparametric estimator based on kernel and a recent alternative the so called OLP estimator that uses a different approach to evaluate the conditional expectation consistently. This analysis allows us extrapolating arbitrage opportunities and relevant information from different markets (futures and options) consistently with the analysis of the underlying. Future research will focus on the extension of those concepts analyzing other possible development and uses of the conditional expectation estimators.

In chapter 5, we examine the use of the conditional expectation in portfolio theory. In particular, we propose three alternative financial applications based on the conditional expectation and a new conditional expectation estimator.

Firstly, we discuss and examine the impact of the correlation matrices and approximation methods in the portfolio theory. In this context, we suggest to approximate the returns using nonparametric regression analysis rather than the classical parametric approach. Using convex dominance testing, we find that the nonparametric regression outperforms its parametric counterpart. Moreover, we show that the dependence measure used to evaluate the joint behavior of returns (stable correlation matrix vs. Pearson correlation matrix) plays a crucial role in the dimensionality reduction of large-scale portfolio problems. For this reason, we propose to use the stable conditional correlation matrix to determine the few factors on which regress the return series and as a regression model the nonparametric ones.

Secondly, we suggest new performance measures that account for the heavy-tailed distribution of the returns. In this context, using stable conditional correlation matrix and the nonparametric techniques, we find that the new suggested methods typically yield the best performance, as measured by the Sharpe ratio and new performance measures.

Finally, we propose a new consistent multivariate kernel estimator of the
conditional expectation and we show that the mean square error in the return approximation is generally lower than the mean square error for other estimators used in literature. Moreover, we deal with the portfolio selection problem from the point of view of different non-satiable investors: namely, risk-averse and risk-seeking. In particular, using a recent returns definition based on the conditional expectation we are able to compare the choices of different investor categories (according to their risk aversion attitude). Therefore, we propose an empirical comparison in which we optimize some classical performances on the returns (according to their new definitions). Thus, even this proposed empirical analysis allows us to evaluate the optimal choices for different categories of investors by using a conditional expectation estimator.

Overall, the thesis contributes the literature in several ways and achieves the general aim. In particular, it allows us to assess the impact and usefulness of the conditional expectation on different financial applications, e.g. arbitrage opportunities, large-scale portfolio selection problems, and optimal portfolio choices.
7 List of References


8 List of Author’s Publications and Research


Publications on Proceeding


Conference without Proceeding

9 Summary

This dissertation examines different financial applications of some conditional expectation estimators. In the first application, we provide some theoretical motivations behind the use of the moving average rule as one of the most popular trading tools among practitioners. In particular, we examine the conditional probability of the price increments and we study how this probability changes over time. In the second application, we present different approaches to evaluate the presence of the arbitrage opportunities in the option market. In particular, we investigate empirically the well-known put-call parity no-arbitrage relation and the state price density. We first measure the violation of the put-call parity as the difference in implied volatilities between call and put options. Furthermore, we propose alternative approaches to estimate the state price density under the classical hypothesis of the Black and Scholes model. In the third application, we investigate the implications for portfolio theory of using conditional expectation estimators. First, we focus on the approximation of the conditional expectation within large-scale portfolio selection problems. In this context, we propose a new consistent multivariate kernel estimator to approximate the conditional expectation. We show how the new estimator can be used for the return approximation of large-scale portfolio problems. Moreover, the proposed estimator optimizes the bandwidth selection of kernel type estimators, solving the classical selection problem. Second, we propose new performance measures based on the conditional expectation that takes into account the heavy tails of the return distributions. Third, we deal with the portfolio selection problem from the point of view of different non-satiable investors, namely risk-averse and risk-seeking investors. In particular, using a well-known ordering classification, we first identify different definitions of returns based on the investors’ preferences. The new definitions of returns are based on the conditional expected value between the random wealth assessed at different times. Finally, for each problem, we propose an empirical application of several admissible portfolio optimization problems using the US stock market.

Key words: Moving Average, conditional probability, systemic risk, arbitrage opportunities, state price density, conditional expectation estimators, and large-scale portfolio selection problems.