Analysis of Steady-State Error in Torque Current Component Control of PMSM Drive

Pavel BRANDSTETTER, Ivo NEBORAK, Martin KUCHAR
Department of Electronics, VSB-Technical University of Ostrava, 70833, Czech Republic
pavel.brandstetter@vsb.cz

Abstract—The paper presents dynamic properties of a vector controlled permanent magnet synchronous motor drive supplied by a voltage source inverter. The paper deals with a control loop for the torque producing stator current. There is shown fundamental mathematical description for the vector control structure of the permanent magnet synchronous motor drive with respect to the current control for d-axis and q-axis of the rotor rotating coordinate system. The derivations of steady-state deviation for schemes with and without decoupling circuits are described for q-axis. The properties of both schemes are verified by MATLAB-SIMULINK program considering a lower and a higher value of inertia and by experimental measurements in our laboratory. The simulation and experimental results are presented and discussed at the end of the paper.

Index Terms—AC motors, electric current control, machine vector control, permanent magnet motors, variable speed drives.

I. INTRODUCTION

At present, the importance of variable speed electrical drives with permanent magnet synchronous motors (PMSM) is growing [1-5]. The magnetic flux and the torque may be controlled by vector control [6, 7], or by direct torque control [8-11]. Current research activities of scientific institutions include a sensorless control of the PMSM drives that uses different rotor speed estimation methods [12-17]. Modern analysis and design methodology of the PMSM enable new construction solutions of the PMSM [18, 19].

In general, it is known that a current control of DC or AC drives has a steady-state error. If the current control loop is designed as the subordinate loop of the speed or position control, the steady-state current error is not such a big problem and it leads to worse dynamic performance of the electric drive. Different situation is for a torque or current control without a superordinate loop, e.g. a torque control of electric drives for robotics or traction applications. In these cases, it is necessary to eliminate or at least to increase the admissible value of the current control error [20-30].

For the vector controlled PMSM, there are very often used circuits to cancel the coupling between flux and torque producing components of the stator current vector. This coupling occurs in the voltage equations expressed in the rotating reference frame for d-axis and q-axis. It is known that the decoupling circuits are necessary for the independent control of both current components of the stator current vector, but in this paper there is shown and verified that the decoupling circuits eliminate the steady-state error of torque producing stator current control too.

II. MATHEMATICAL MODEL OF THE VECTOR CONTROLLED PERMANENT MAGNET SYNCHRONOUS MOTOR

For assembling a mathematical model of the PMSM, we can use theory of a general AC machine. Given that the excitation magnetic flux of the rotor is generated by the permanent magnets, for describing the properties of the machine and for the vector control it is suitable to choose as reference the [d, q] rotating coordinate system. The magnetic flux generated by the permanent magnets induces in the stator winding a voltage during rotation of the PMSM rotor.

The vector control of the PMSM uses a principle of stator current vector components separation. These current components \( i_{sd} \) and \( i_{sq} \) are orthogonal and influence a magnetization and torque of the PMSM.

The d-axis of the [d, q] rotating coordinate system is determined by the position of the magnetic flux vector created by permanent magnets \( \Phi_F \) (Fig. 1).

For the stator voltage vector components in the [d, q] rotor rotating coordinate system, there are valid the following voltage equations:

\[
\begin{align*}
    u_{sd} &= R_{sd} i_{sd} + \frac{d\Psi_{sd}}{dt} - \omega \Psi_{sq} = \\
    &= R_{sd} i_{sd} + \left( L_{sd} \frac{d i_{sd}}{dt} \right) + \Phi_F - \omega L_{sq} i_{sq} = \\
    &= R_{sd} i_{sd} + L_{sd} \frac{d i_{sd}}{dt} - \omega L_{sq} i_{sq} ,
\end{align*}
\]

\[
\begin{align*}
    u_{sq} &= R_{sq} i_{sq} + \frac{d\Psi_{sq}}{dt} + \omega \Psi_{sd} = R_{sq} i_{sq} + L_{sq} \frac{d i_{sq}}{dt} + \\
    &+ \omega \left( L_{sd} i_{sd} + \Phi_F \right) ,
\end{align*}
\]

where: \( i_{sd}, i_{sq} \) – stator current vector components in [d, q] rotating coordinate system; \( u_{sd}, u_{sq} \) – stator voltage vector components; \( L_{sd}, L_{sq} \) – stator inductance components; \( \Phi_F \) – magnetization of permanent magnets.
components in \([d, q]\) rotating coordinate system; \(\Phi_F\) – magnetic flux of permanent magnets; \(L_{sd}\) – stator inductance in \(d\)-axis; \(L_{sq}\) – stator inductance in \(q\)-axis; \(R_S\) – stator phase resistance; \(\omega\) – electrical angular speed of the rotor; \(\Theta\) – rotor angle.

The PMSM torque is defined as follows:

\[
T = \frac{3}{2} p \left[ \Phi_F + \left( L_{sd} - L_{sq} \right) i_{sd} \right] i_{sq} .
\]  

(3)

For a Surface PMSM, \(L_{sd} \approx L_{sq}\), and then Eq. (3) will have the form:

\[
T = \frac{3}{2} p \Phi_F i_{sq} .
\]  

(4)

The motion equation is:

\[
T - T_L = J_m \frac{d\Omega_m}{dt} ,
\]  

where: \(J_m\) – inertia; \(p\) – number of pole pairs; \(T\) – PMSM torque; \(T_L\) – load torque; \(\Omega_m\) – mechanical rotor angular speed.

The electrical rotor speed is given by the derivative of the electrical rotor angle:

\[
\omega = p \Omega_m = \frac{d\Theta}{dt} ,
\]  

(6)

Electromagnetic time constants in \(d\)-axis and \(q\)-axis are defined by following equations:

\[
T_{sd} = \frac{L_{sd}}{R_S} ,
\]  

(7)

\[
T_{sq} = \frac{L_{sq}}{R_S} .
\]  

(8)

From voltage Eqs. (1) and (2), we obtain the equations for the current components that are important for further analysis. After Laplace transformation, we obtained the following relationships for the \(d\)- and \(q\)-components of the stator current vector.

\[
i_{sd} = \frac{1}{R_S \left( 1 + s T_{sd} \right)} \left[ u_{sd} + \omega L_{sd} i_{sd} \right] ,
\]  

(9)

\[
i_{sq} = \frac{1}{R_S \left( 1 + s T_{sq} \right)} \left[ u_{sq} - \omega \left( \Phi_F + L_{sq} i_{sd} \right) \right] .
\]  

(10)

The square brackets in Eqs. (9) and (10) contain terms that represent the undesirable coupling between \(d\) and \(q\) components.

### III. CONTROL STRUCTURE OF THE PMSM DRIVE

The speed control structure of the permanent magnet synchronous motor drive with the vector control is shown in Fig. 2. The control structure is formed by so-called subordinate control loops that consist of one or more simple controllers. In Fig. 2, the following blocks were used:

- **SM** – synchronous motor;
- **FC** – frequency converter;
- **T 3/2** – block of the Clarke transformation;
- **PWM** – block of the pulse-width modulation;
- **ESP** – block of the position estimation;
- **CS** – current sensors;
- **DEC** – block of the decoupling of \(d\)- and \(q\)-axes;
- **e^{j\Theta}, e^{-j\Theta}\) – blocks for rotation of vectors;
- **TAB** – block for calculating the sin/cos functions.

In the sensorless control technique, a position or speed estimator which uses different types of estimation methods is used. We used a position sensor PS for the estimation of the rotor angle \(\Theta\).

---

**Figure 2. Speed control structure of the vector controlled AC drive with the PMSM**
The vector rotation and the reverse vector rotation of the complex space vector components from the \([\alpha, \beta]\) stationary coordinate system to the \([d, q]\) rotor rotating coordinate system respectively, are performed using the rotor angle \(\Theta\).

The stator current component \(i_{Sdref}\) that produces the reference flux is zero for a speed up the nominal value \(\Omega_{mN}\). Above this value of the speed, this component of the stator current decreases to negative values according to the actual value of the speed \(\Omega_m\). This is the so-called field weakening of the PMSM.

The stator current component \(i_{SdRef}\) that produces the reference torque is determined by the PI speed controller. Both components of the stator current vector are then controlled in subordinate current control loops.

On the basis of previous equations, it can be drawn a current control block scheme of the vector controlled PMSM without decoupling (Fig. 3).

Fig. 3 consists of the following transfer functions:
- \(F_{CCd}\) – transfer function of the current controller in \(d\)-axis;
- \(F_{CCq}\) – transfer function of the current controller in \(q\)-axis;
- \(F_{FC}\) – transfer function of the frequency converter;
- \(F_{CS}\) – transfer function of the current sensor.

Fig. 3 describes the coupling between the stator current component \(i_{Sd}\) that produces the magnetizing current and the stator current component \(i_{Sq}\) that produces the torque, necessary for the vector control of the PMSM.

The coupling cancellation, that we called decoupling, can be made using members whose output signals are added to output signals of the current controllers.

The voltage components \(u_{kd}\) and \(u_{kq}\) are added to the output voltage signals of the current controllers to eliminate the coupling between \(d\)-axis and \(q\)-axis according to (11) and (12). These components are determined in block of decoupling DEC (Fig. 2).

The decoupling is performed using the following equations:

\[
\begin{align*}
    u_{kd} &= -\omega L_{sq} i_{sq}, \\
    u_{kq} &= \omega (\Phi_F + L_{sd} i_{sd}).
\end{align*}
\]

The Chapter IV deals with the torque current component control only. For the analysis of the decoupling influence, it is considered an activity of the PMSM drive in the speed range to the nominal speed \((i_{kd} = 0)\).

### IV. PARAMETERS OF PMSM DRIVE

For the analysis of the steady-state error of the torque current component, the PMSM drive from the laboratory of the Department of Electronics, Technical University of Ostrava is considered.

In Table I, the main parameters of the used Surface PMSM with rare earth PM type 1FK 7063-5AF71 (Siemens) are shown. The induced voltage is approximately sinusoidal. The PMSM is loaded by an induction machine. The inertia of the PMSM drive is \(J_t = 31.1 \times 10^{-4} \text{kgm}^2\).

#### Table I. Parameters of Surface-Mounted PMSM

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Quantity</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(P_N)</td>
<td>nominal power</td>
<td>2.29 kW</td>
</tr>
<tr>
<td>(T_N)</td>
<td>nominal torque</td>
<td>7.3 Nm</td>
</tr>
<tr>
<td>(n_N)</td>
<td>nominal speed</td>
<td>3000 rpm at (f = 200) Hz</td>
</tr>
<tr>
<td>(n_{max})</td>
<td>maximal speed</td>
<td>7200 rpm</td>
</tr>
<tr>
<td>(U_{pan})</td>
<td>induced line-to-line voltage</td>
<td>263 V at 3000 rpm</td>
</tr>
<tr>
<td>(I_{SN})</td>
<td>nominal stator current</td>
<td>5.6 A</td>
</tr>
<tr>
<td>(\Phi_F)</td>
<td>magnetic flux of the PM</td>
<td>0.1706 Wb</td>
</tr>
<tr>
<td>(P)</td>
<td>number of pole pairs</td>
<td>4</td>
</tr>
<tr>
<td>(R_s)</td>
<td>stator resistance</td>
<td>0.65 (\Omega)</td>
</tr>
<tr>
<td>(L_{sd} = L_{sq} = L_s)</td>
<td>stator inductance</td>
<td>7.7 mH</td>
</tr>
<tr>
<td>(J_m)</td>
<td>motor inertia</td>
<td>0.00151 (\text{kgm}^2)</td>
</tr>
</tbody>
</table>

**Figure 3.** Speed control structure of the vector controlled AC drive with the PMSM

![Figure 3](https://example.com/figure3.png)
The PMSM was supplied by a frequency converter (FC) with the DC link voltage $U_d = 200$ V. The control of the FC output voltage is performed by means of a sine PWM with the frequency $f_p = 10$ kHz and the voltage $U_{pmax} = \pm 1$ V.

The transfer function of the frequency converter is indicated as $F_{FC}$ in Eq. (14). The gain and the time constant of the FC are $K_{FC} = 100$, and $T_{FC} = 0.05$ ms, respectively.

The current sensor is described by the transfer function $F_{CS}$ in Eq. (14). The gain and the time constant of the current sensor are $K_{CS} = 1$ A/A, and $T_{C} = 0.025$ ms, respectively.

The current controller in the $q$-axis is described by the transfer function $F_{CCq}$ in Eq. (14). The gain and the time constant of the current controller are $K_{CCq} = 0.609$, and $T_{CCq} = 11.8$ ms, respectively. The current controller parameters were obtained by the optimal modulus method.

The rotor speed and position are measured using the incremental sensor IRC 2048 with the four times multiplication of pulses. The resulting number of pulses is 8192 per revolution. The sampling period is $T_s = 5$ ms.

The time constant of the speed sensor is $T_{SS} = 2.5$ ms. The speed sensor gain $K_{SS}$ is not reflected in the following calculations. The transfer function of the speed sensor is indicated as $F_{SS}$ in Eq. (20).

For the analysis, a regime with the nominal excitation of the steady-state PMSM at nominal speed and therefore at $i_{sd} = i_{sdref} = 0$ is considered. The influence of the load torque $T_L$ is neglected.

V. TORQUE CURRENT COMPONENT CONTROL WITHOUT DECOUPLING

The disadvantage of the control structures in Figs. 4 and 5 is that they exhibit the current control deviation (control error) from the viewpoint of the control variable, i.e. the reference current.

The steady-state control error is calculated by:

$$
\Delta I_{sqref} = \lim_{s \to 0} \left( s \frac{1}{1 + F_0} \frac{I_{sqref}}{s} \right) = \frac{1}{1 + K_0} I_{sqref} - I_{sqref} = \frac{1}{1 + 22.98} I_{sqref} = 0.0417 I_{sqref}
$$

As it can be observed in (13), for the considered parameters, the relative steady-state error is 4.17%.

According to Fig. 5, the transfer function of the open control loop for the torque current component is described by:

$$
F_0 = \frac{I_{sq}}{\Delta I_{sq}} = F_{CCq} F_{FC} F_{Mq} \frac{s I_{d}'}{3/2} p \Phi F
$$

$$
F_0 = K_{CCq} \frac{(1 + s T_{CCq})}{s T_{CCq} F_{FC}} \frac{s J_q}{1 + s T_q} K_{CS}
$$

$$
1/ \left( p \Phi F \right) s J_q \frac{K_{CS}}{(1 + s T_{FC})(1 + s T_{CS})} = \frac{K_0}{(1 + s T_{FC}) (1 + s T_{CS})} = \frac{K_{CCq} K_{FC} J_{KL} K_{CS}}{T_{CCq} (3/2) (p \Phi F)^2}
$$

$$
= 0.609 \cdot 100 \cdot 3.11 \cdot 10^{-4} \cdot 1
$$

$$
= 0.0118 \cdot (3/2) \cdot (4 - 0.1706)^2 = 22.98
$$

In Eq. (14), the term $F_{Mq}$ represents the transfer function of the PMSM in the $q$-axis, defined as:

$$
F_{Mq} = \frac{\Omega_m}{U_{Lq}} = \frac{1/ \left( p \Phi F \right)}{1 + s T_m + s^2 T_m T_s}
$$

In Eq. (17), the term $T_m$ can be regarded as a mechanical time constant and the term $T_s$ can be regarded as an electromagnetic motor time constant.

$$
T_m = \frac{2 J_{RL} R_s}{3 p^2 \Phi F^2} = 2.311 \cdot 10^{-4} \cdot 0.65 = 0.0029 \text{ s}
$$

$$
T_s = \frac{L_S}{R_S} = \frac{0.0077}{0.65} = 0.0119 \text{ s}
$$

VI. TORQUE CURRENT COMPONENT CONTROL WITH DECOUPLING

The block scheme and the adjusted block scheme of the torque current component control with decoupling are shown in Figs. 6 and 7.
The transfer function of the open control loop with the decoupling according to Figs. 6 and Fig. 7 is defined by:

\[ F_0 = \frac{I_{SQ}}{\Delta I_{SQ}} = \frac{I_{SQ}}{I_{SQref} - I_{SQ}} = \frac{F_{CCq}F_{MC}}{1 - \frac{F_{CCq}F_{MC}P\Phi_F}{K_{SS}K_{FC}} \left(3/2\right) p\Phi_F F_{CS}}, \tag{20} \]

In Eq. (21), the term \( T_m \) can be regarded as a mechanical time constant.

\[ F_0 = K_{CCq} \frac{1 + sT_{CCq}}{sT_{CCq}} \frac{K_{FC}}{1 + sT_{FC}} \frac{1/p\Phi_F}{\left(1 + sT_{FC}\right)\left(1 + sT_w + s^2T_wT_{SS}\right)} \frac{p\Phi_F}{\left(1 + sT_w + s^2T_wT_{SS}\right)K_{SS}K_{FC}} \left(3/2\right) p\Phi_F \frac{K_{CS}}{1 + sT_{CS}}. \tag{21} \]

The steady-state error at the step of the reference value \( I_{SQref} \) is defined as follows:

\[ \Delta I_{SQref} = \lim_{s \to 0} \left( s \frac{1}{1 + F_0} \frac{I_{SQref}}{s} \right) = 0. \tag{22} \]

It can be seen, that at steady-state, the error is zero.

VII. SIMULATION RESULTS

The structures of the variable speed electric drives are very complex, therefore it is very suitable to use appropriate simulation tools for verifying these structures. According to the block schemes in Figs. 4 and 6, respectively, we created MATLAB-Simulink simulation structures.

Figs. 8 and 9 show simulated waveforms of the reference torque current component, actual torque current component, and PMSM speed. These simulated waveforms confirm the derived control deviations mentioned in Chapter V. For example in simulation, the steady-state control error of the torque current component is \( \Delta I_{SQ} = 0.0838 \) A at inertia \( J_t = 0.00311 \) kgm\(^2\). According to Eq. (13), the steady-state control error is \( \Delta I_{SQ} = 0.0834 \) A.

The waveforms of the actual torque current component \( i_{SQ} \) were obtained from the control structure without decoupling (Figs. 8 and 9) and with decoupling (Figs. 10 and 11) at the step of the reference torque current \( I_{SQref} = 2 \) A.

The control structure without decoupling gives at steady-state, for the lower inertia \( J_t = J_m = 0.00151 \) kgm\(^2\) (Fig. 8), the relative error:

\[ \delta_{ISq} = \frac{\Delta I_{SQ}}{I_{SQref}} \times 100 = 2 - 1.8349 \times 100 = 8.26 \%, \tag{23} \]

what is in accordance with Eq. (13).

The control structure without decoupling gives at steady-state, for inertia \( J_t = J_m = 0.00311 \) kgm\(^2\), and steady-state value of torque current component of 1.8349 A.

The control structure without decoupling gives at steady-state, for inertia \( J_t = J_m = 0.00151 \) kgm\(^2\) (Fig. 9), the relative error:

\[ \delta_{ISq} = \frac{\Delta I_{SQ}}{I_{SQref}} \times 100 = 2 - 1.9162 \times 100 = 4.19 \%, \tag{24} \]

what is in accordance with Eqs. (13) and (15).

The small difference appears since in steady-state, the simulated waveform of the torque current component is not still.

According to Eq. (15), for the reduced value of inertia, it is possible to obtain the open loop gain of \( K_0 = 11.16 \) against to the original value \( K_0 = 22.98 \). In this case, according to Eq. (13), at steady-state, the error is defined by:

\[ \Delta I_{SQ} = \frac{1}{1 + K_0} \frac{I_{SQref}}{1} = \frac{1}{1 + 11.16} \frac{I_{SQref}}{1} = 0.08225I_{SQref}. \tag{25} \]

According to Eq. (25), at steady state, the relative error is 8.22%.

At the control structure with decoupling, at steady-state the relative error is equal zero (Fig. 10), that is in accordance with Eq. (22). For the lower value of inertia \( J_t = J_m = 0.00151 \) kgm\(^2\), the differences are not so evident as
for the structure without decoupling.

Figs. 11 and 12 show the waveforms of the speed during acceleration of the AC drive with the currents as is shown in Fig. 8 - Fig. 10 without load.

The details plotted in Fig. 8 - 12 confirm the mentioned theoretical assumptions. It is obvious that the vector controlled AC drive with the decoupling has better dynamics, i.e. faster speed growth.

VIII. EXPERIMENTAL RESULTS

For the experimental verification of the steady-state deviation of the torque current component in the structure of the vector controlled PMSM, experimental measurements were performed on the PMSM drive from the laboratory of the Department of Electronics, VSB - Technical University of Ostrava, with a PMSM type 1FK 7063-5AF71 (Siemens). The PMSM was connected with an induction machine as load, which increases the moment of inertia of the whole set (Fig. 13).

The PMSM parameters used in the simulation correspond to these of the real PMSM drive (Chapter IV). The PMSM was powered by a frequency converter that consists of a rectifier, voltage DC link and a voltage source inverter. The control system contains a TMS320F28335 DSP.

The algorithm including the stator currents and DC-link voltage measuring is processed with the sampling period of 50 μs.

Figs. 16 and 17 show waveforms of selected quantities during the start-up of the PMSM drive to speed 600 rpm without decoupling (Fig. 16) and with decoupling (Fig. 17). The PMSM drive is without load.
The described analysis demonstrates that at steady-state the error of the torque component of the stator current vector is eliminated by involving correction decoupling circuits. This fact was derived in (22) and verified by the simulation and experimental results.

However, the practical accuracy depends on settings in the control block. Deviations could occur due to change in the gain of the frequency converter.

For the control structure without decoupling circuits, the steady state error greatly depends on system and controller parameters, as it can be seen on Eqs. (13) and (15).

If the PMSM drive works in the field weakening mode, where the magnetizing current is decreased from zero to negative values, it is necessary to carry out a gain adaptation in the decoupling circuit with respect to the change of the magnetizing current component.

The above-mentioned conclusions, with some modifications, are also valid for other AC drives with vector control or with similar quality control and of course for DC drives.

Measured experimental waveforms confirmed the theoretical assumptions and the simulation results. In the case of the control structure without decoupling circuit, when the inertia is 0.00311 kgm², the steady state value of the torque current component was of 1.9162 A in simulation and 1.9 A in the experiment. In the case of the control structure with decoupling circuit, at steady-state, the current error is zero in both cases, simulation and experiment.

REFERENCES


