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INTERNAL GEARING WITH TEETH NUMBER Z₁ = |Z₂|

VNITŘNÍ OZUBENÍ S POČTEM ZUBŮ Z₁ = |Z₂|

Abstrakt
Každý návrh vnitřního ozubení s malým rozdílem počtu zubů mezi pastorkem a korunovým kolem je komplikovaný. Čím menší rozdíl počtu zubů je využitý, tím větší množství problémů a omezení je zapotřebí řešit. Tyto řešení a postupy můžeme využívat až do rozdílu počtu zubů |z₂| - |z₁| = 1 (počet zubů vnitřního ozubení musí být uvažován záporný). Tato řešení však neplatí pro stejný počet zubů. Je třeba použít zcela jiné postupy – ale řešení existuje.

Abstract
Each design of internal gearings with small difference between number of teeth for annular gear and pinion is complicated. The smaller difference of the teeth numbers is used the greater amount of problems and restrictions is necessary to solve. These solutions and procedures we can use until the teeth difference |z₂| - |z₁| = 1 (number of teeth for internal gearing must be taken as a negative number). But for equal teeth number these solutions are not valid. It is necessary to use absolutely different procedures – but a solution exists.

Used symbols
a pitch centre distance
aₜ working centre distance
dₜ base diameter of pinion
dₙ base diameter of ring gear
eₙ base spacewidth of ring gear
mₙ normal module
sₜₜ tooth thickness on the base cylinder of pinion
x₁ addendum modification coefficient of pinion
x₂ addendum modification coefficient of ring gear
z₁ number of teeth of pinion
z₂ number of teeth of ring gear
αₙ basic rack flank angle
αₜ transverse rack flank angle
αₜₜ working transverse pressure angle
Δz difference between number of teeth

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Working transverse pressure angle

For internal gearing this angle is growing up proportionally to decreasing of the teeth number difference between a ring gear and a pinion \( \Delta z = |z_2| - z_1 \) (fig. 1). Basic circles \( d_{b1} \) and \( d_{b2} \) have the same diameters for \( \Delta z = 0 \). For N gearings (both gears are without addendum modification coefficient) they have also common centre.

\[
\begin{align*}
z_1 &= 25 \quad z_2 = -35 \\
x_1 &= 0 \quad x_2 = -3 \\
\alpha_{rw} &= 46^\circ
\end{align*}
\]

\[
\begin{align*}
z_1 &= 32 \quad z_2 = -35 \\
x_1 &= 0 \quad x_2 = -3 \\
\alpha_{rw} &= 61^\circ
\end{align*}
\]

\[
\begin{align*}
z_1 &= 35 \quad z_2 = -35 \\
x_1 &= 0 \quad x_2 = -3 \\
\alpha_{rw} &= 90^\circ
\end{align*}
\]

Fig. 1 - lines of action

The gearing becomes itself a co-axial clutch with involute splining and with backlash-free manufacturing are all teeth in contact. We can say that it is not a usual mesh. A usual mesh occurs after shifting of basic circles centers out of common point. The backlash will be eliminated in the vertical direction by this shifting. Working pressure angle stays for any shifting always 90°. Unfortunately it is impossible to calculate working centre distance \( a_w \) using usual relations:

\[
a_w = a \cdot \frac{\cos \alpha_r}{\cos \alpha_{rw}}
\]  

(1)

Because for \( \Delta z = 0 \) is valid \( a = 0 \) and \( \cos \alpha_{rw} = \cos 90^\circ = 0 \). So we get an indeterminate expression zero divided by zero and thus it is impossible to find a solution. Action line is namely parallel to a connection of gears centers (fig. 1).

From the basic relation of V gearing (2) is valid that working transverse pressure angle \( \alpha_{rw} \) will be always 90°. This angle is independent on the sum of addendum modification coefficients. It is because \( z_2 + z_1 = 0 \).

\[
\begin{align*}
\operatorname{inv} \alpha_{rw} &= \frac{2 \cdot (x_1 + x_2)}{z_1 + z_2} \cdot \tan \alpha_r + \operatorname{inv} \alpha_r
\end{align*}
\]

(2)

The correct output we will gain from a condition of an eliminating of a backlash. The pinion must shift vertically up itself for a size \( 0.5 \cdot (e_{b2} - s_{b1}) \). This shifting then creates working centre distance \( a_w \).

\[
a_w = 0.5 \cdot (e_{b2} - s_{b1})
\]  

(3)

For relevant width and space on the basic circles are valid:

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\[
\begin{align*}
    s_{s1} &= d_{s1} \cdot \left( \frac{\pi}{2 \cdot z_1} + \frac{2 \cdot x_1 \cdot \tan \alpha}{z_1} + \text{inv} \alpha \right) \\
    e_{s1} &= |d_{s1}| \cdot \left( \frac{\pi}{2 \cdot |z_1|} - \frac{2 \cdot x_2 \cdot \tan \alpha}{|z_2|} + \text{inv} \alpha \right)
\end{align*}
\]

After substituting into relation (1) and considering that \(z_1 = |z_2|\), we will get:

\[
e_{s1} - s_{s1} = 2 \cdot m_n \cdot \sin \alpha \cdot (x_1 + x_2)
\]

And now it is easy to get the final result – working centre distance \(a_w\). It is one half of the backlash for both side of the tooth:

\[
a_w = m_n \cdot \sin \alpha \cdot (x_1 + x_2)
\]

On the basis of this relation (7) it pays that the sum \(x_1 + x_2\) must be always a negative one (teeth must be thinner). Calculation of all rest dimensions and meshing parameters of the gearing is the same like for common internal gearing. The detailed figure of meshing area is on the figure 2.

![Fig. 2 – meshing area](image)

![Fig. 3 - full view](image)

**Conclusion**

We can imagine a practical utilization such a gearing in a row of applications which require small eccentricity and identical revolutions. For example a coupling of a pair of co-axial shafts. Internal gearing with \(\Delta z = 0\) serves here as a coupling. Its advantage is securing of identical revolutions of
both shafts, which is not common at this types of couplings. Moreover using gearing as an eccentric coupling gives a higher transmission efficiency.

**Literature**


**Oponent:** prof. Ing. Horst Gondek, DrSc.