CONVENTIONAL CONTROLLER TUNING FOR MONOTONE SELF-REGULATING PLANTS WITH A TIME DELAY

SEŘÍZENÍ KONVEČNÍCH REGULÁTORŮ PRO MONOTÔNNÍ REGULOVANÉ SOUSTAVY S DOPRAVNÍM ZPOŽDĚNÍM

Abstract:
The paper describes an original analytic-experimental approach to simple digital and analog conventional PI and PID controller tuning required for monotone self-regulating plants with a time delay. The values of adjustable controller parameters ensuring a marginal monotone control process are determined analytically, and the gain of the controller is tuned experimentally so that an overshoot of a step response of the control system corresponds to the concept of the designer. The approach is based on the desired model method, with its use being exemplified.

1 Introduction

There exist a number of different PI and PID controller tuning methods. In most cases, however, they do not provide unified approach on digital and analog controllers. In addition, they are complex and do not belong to any theory. The methods are described in detail, for example in Refs (Åström & Hägglund 1995; Shinskey 1996).

This paper is devoted to a new analytic-experimental approach based on the desired model method (inverse dynamics method), which enables simple and fast tuning of digital and analog PI and PID controllers employed in monotone self-regulating plants with a time delay (Vítečková 1996; Vítečková, Víteček & Smutný 2000).

2 Controlled Plants

The proposed approach assumes that the transfer function of the monotone self-regulating plant can be approximated by either of the transfer functions

\[ G_p(s) = \frac{k_1}{T_1 s + 1} e^{-T_{tr \ast}} \]  

\[ G_p(s) = \frac{k_1}{(T_1 s + 1)^n} e^{-T_{tr \ast}} \]  

where \( k_1 \) is the plant gain, \( T_1 \) – the plant time constant, \( T_{tr} \) – the plant time delay, \( n \) – the plant order (\( n = 1, 2 \)), \( s \) – the complex variable in Laplace transform.
The time constant $T_1$ and the time delay $T_{d1}$ in transfer function (1) can be obtained directly from the step response of the filtered or otherwise adapted controlled plant on the basis of the relations

$$T_1 = 1.245(t_{0.7} - t_{0.33}),$$
$$T_{d1} = 1.498t_{0.33} - 0.498t_{0.7}.\quad (3)$$

Likewise, the time constant $T_2$ and the time delay $T_{d2}$ in transfer function (2) can be determined on the basis of the relations

$$T_2 = 0.794(t_{0.7} - t_{0.33}),$$
$$T_{d2} = 1.937t_{0.33} - 0.937t_{0.7}.\quad (4)$$

Relations (3) were obtained analytically while relations (4) were derived numerically from the coincidence of the approximate step response with the real step response of the controlled plant with the following values $y(0) = 0, y(t_{0.33}) = 0.33y(\infty), y(t_{0.7}) = 0.7y(\infty)$ and $y(\infty)$.

For experimental identification of monotone self-regulating plants of a higher order the Strejc’s method is often used. This leads to a transfer function with the same time constants and time delay

$$G_p(s) = \frac{k_1}{(T_1s + 1)^s} e^{-T_{d1}s}.\quad (5)$$

Transfer function (5) can be converted into forms (1) and (2) with the help of Tab. 1, as can be seen in Example 1 (Vitečková 1996; Vitečková 1998)

$$\frac{1}{(T_1s + 1)^s} e^{-T_{d1}s} \quad \leftrightarrow \quad \frac{1}{T_1s + 1} e^{-T_{d1}s} \quad \leftrightarrow \quad \frac{1}{(T_2s + 1)^s} e^{-T_{d2}s}.\quad (6)$$

Table 1 was obtained numerically for the coincidence of the step responses of the models with the following values: $y(0), y(t_{0.33}), y(t_{0.7})$ and $y(\infty)$.

**Example 1**

The transfer function

$$G_p(s) = \frac{1}{(1.5s + 1)^s} e^{-3s}$$

needs to be transformed into forms (1) and (2). The time constants and time delay are in seconds.

**Tab. 1 Table for the transformation of transfer functions in accordance with scheme (6)**

<table>
<thead>
<tr>
<th>$\frac{1}{(T_1s+1)} e^{-T_{d1}s}$</th>
<th>$n$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{1}{T_1s+1} e^{-T_{d1}s}$</td>
<td>$\frac{T_1}{T_1}$</td>
<td>1</td>
<td>1.568</td>
<td>1.980</td>
<td>2.320</td>
<td>2.615</td>
<td>2.881</td>
</tr>
<tr>
<td>$\frac{T_{d1} - T_{d2}}{T_1}$</td>
<td>0</td>
<td>0.552</td>
<td>1.232</td>
<td>1.969</td>
<td>2.741</td>
<td>3.537</td>
<td></td>
</tr>
<tr>
<td>$\frac{T_{d1} - T_{d2}}{T_1}$</td>
<td>0.638</td>
<td>1</td>
<td>1.263</td>
<td>1.480</td>
<td>1.668</td>
<td>1.838</td>
<td></td>
</tr>
<tr>
<td>$\frac{T_{d2}}{T_1}$</td>
<td>$-0.352^*$</td>
<td>0</td>
<td>0.535</td>
<td>1.153</td>
<td>1.821</td>
<td>2.523</td>
<td></td>
</tr>
</tbody>
</table>

* Applicable for $T_{d1} > 0.352T_1$.
Solution:

For transfer function (7) it holds: \( k_1 = 1, n = 4, T_4 = 1.5 \) and \( T_{d4} = 5 \).

Therefore, for \( n = 4 \), column 4 in Tab. 1 will be used.

a) \( \frac{T_1}{T_4} = 2.320 \Rightarrow T_1 = 3.48, \quad \frac{T_{d4} - T_{d4}}{T_4} = 1.969 \Rightarrow T_{d1} \approx 7.96 \)

\[
G_p(s) = \frac{1}{3.48s + 1} e^{-7.96s} \tag{8}
\]

b) \( \frac{T_1}{T_4} = 1.480 \Rightarrow T_2 = 2.22, \quad \frac{T_{d3} - T_{d4}}{T_4} = 1.153 \Rightarrow T_{d2} \approx 6.73 \)

\[
G_p(s) = \frac{1}{(2.22s + i)^2} e^{-6.73s} \tag{9}
\]

The plant and its approximations step responses are shown in Fig. 1.

3 Synthesis of the Control System

Consider an ordinary closed-loop control system, where the transfer function of the controller \( G_C \) and the transfer function of the controlled plant \( G_p \) have forms consistent with the data given in Table 2.

The synthesis of the feedback control system on the basis of the desired model method (dynamics inversion method) lies in the determination of the controller with the transfer function \( G_C \), which ensures the desired transfer function of the closed-loop control system \( G_{sys} \), i.e.

\[
G_C = \frac{1}{G_p\left(1 - G_{my}\right)}, \quad G_{my} = \frac{Y}{W} \tag{10}
\]

Next, we consider a control system with a digital controller with the transfer function \( G_C(z) \), where the D/A converter corresponding to the sampler and the zero-order hold is used. Therefore for the discrete transfer function of the controlled plant \( G_p(z) \), it is possible to write

\[
G_p(z) = \frac{z^{-1}}{z} Z\left\{ L^1\left\{ \frac{G_p(s)}{s} \right\} \right\}_{z \to s} \tag{11}
\]

where \( k \) is the relative discrete time.

It is supposed that the desired transfer function of the feedback control system \( G_{my}(z) \) has the form (Vítcěková 1996; Vítcěková 1998)

\[
G_{my}(z) = \frac{k_d T}{z - 1 + k_d T z^{-d} - 1} z^{-d_i} \tag{12}
\]

\[
d_i = \frac{T_i}{T}, \quad i = 1, 2 \tag{13}
\]

where \( k_d \) is the gain of the open-loop control system with the digital controller, \( d_i \) — the relative discrete time delay (for simplicity, only the integer number is considered).
Tab. 2 Transfer functions of the controlled plants and controllers

<table>
<thead>
<tr>
<th>CONTROLLED PLANT $G_p(s)$</th>
<th>TYPE</th>
<th>ANALOG CONTROLLER $G_c(s)$</th>
<th>DIGITAL CONTROLLER $G_c(z)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{k_1}{T_is+1}e^{-\tau_{es}}$</td>
<td>PI</td>
<td>$k_r\left(1+\frac{1}{T_is}\right)$</td>
<td>$k_r\left(1+\frac{T}{T_is}z\right)$</td>
</tr>
<tr>
<td>$\frac{k_1}{(T_is+1)^2}e^{-\tau_{es}}$</td>
<td>PID</td>
<td>$k_r\left(1+\frac{1}{T_is}+T_0s\right)$</td>
<td>$k_r\left(1+\frac{T}{T_is}z\right)\frac{z-1}{z+\frac{T_0}{T}}$</td>
</tr>
</tbody>
</table>

Then after the discretization of the transfer function of the controlled plant (1) by means of relation (11) and on the basis of relations (10) and (12), the formulas

$$G_c(z) = k_r\left(1+\frac{T}{T_is}z\right)$$  \hspace{1cm} (14)

$$T_i = \frac{c_i}{1-c_1}, \hspace{1cm} c_i = e^{-\frac{T_i}{c_1}}$$  \hspace{1cm} (15)

$$k_r = \frac{k_r c_i T_i}{k_i (1-c_1)} = \frac{k_r T_i}{k_i}$$  \hspace{1cm} (16)

are obtained.

From this form of transfer function (14) it is obvious that it is the digital PI controller (see Table 2).

Likewise, after the discretization of the transfer function of the controlled plant (2) by means of relation (11) and on the basis of relations (10) and (12) the formulas

$$G_c(z) = k_r(z)\left(1+\frac{T}{T_is}z+\frac{T_0}{T}z-1\right)$$  \hspace{1cm} (17)

$$T_i = \frac{2c_i}{1-c_1}, \hspace{1cm} c_i = e^{-\frac{T}{c_1}}$$  \hspace{1cm} (18)

$$T_0 = \frac{c_2}{2(1-c_1)} T = 0.25T_i$$  \hspace{1cm} (19)

$$k_r(z) = \frac{2k_r (1-c_1)c_2 T}{k_i (Az+B)}$$  \hspace{1cm} (20)

are obtained.

In this case, the form of transfer function (17) shows the digital PID controller (see Table 2). Two significant problems appear as a result. The controller gain $k_r(z)$ is a function of the complex variable $z$, which does not permit using the conventional digital PID controller. It implies the occurrence of
inadmissible oscillation of the manipulated variable too. Both these problems can be removed by the use of the constant gain (Vitečková 1996; Vitečková 1998)

\[ k_\sigma = \lim_{z \to 0} k_\sigma(z). \]  

(21)

which keeps the coincidence only in steady state. On the basis of the relation (21) resulting from (20) we obtain

\[ k_\sigma = \frac{2k_c c_z T}{k_i (1 - c_1)} = \frac{k_c T_i}{k_i}. \]  

(22)

Now the gain of the open-loop control system with the digital controller \( k_\sigma^* \), which ensures the marginal monotone control process, will be determined. At least one stable double real root of the characteristic polynomial [see the transfer function of the control system (12)]

\[ N(z) = z^{d_1} - z^{d_2} + k_\sigma T \]  

(23)
corresponds to the marginal monotone control process. From the relations

\[ N(z) = 0, \quad \frac{dN(z)}{dz} = 0 \]  

(24)

the stable double real root

\[ z^{*}_{m} = \frac{d_1}{d_1 + 1} \]  

(25)
is obtained, which corresponds to the open-loop control system gain

\[ k_\sigma^* = \frac{1}{T} \left( \frac{d_1}{d_1 + 1} \right) \]  

(26)

which ensures the marginal monotone control process.

The relation for the gain (26) can be simplified by using the approximation [see (13)]

\[ k_\sigma^* = \frac{1}{T} \left( \frac{d_1}{d_1 + 1} \right) \approx \frac{1}{(4 - e)T + eT_{\alpha}} \]  

(27)

with an error less than 0.5% for \( d_1 > 0.5 \) (Vitečková 1998).

For routine practical use the terms \( c_1 \) and \( c_2 \) in the relations for the determination of the adjustable parameters of the controllers are inappropriate. After using the approximation

\[ e^x \approx \left( \frac{1 - \frac{x}{2}}{2} \right) \left( \frac{1 + \frac{x}{2}}{2} \right)^{-1} \]  

(28)

relations (15) and (18) were simplified and, therefore, for computation of the values of the adjustable parameters, which ensure the marginal monotone control process, it is possible to use the following relations:

**PI controller** [for the transfer function of the controlled plant (1)]

\[ T_i^* = T_i - 0.5T, \quad k_\sigma^* \approx \frac{T_i^*}{k_i (1.28T + 2.72T_{\alpha})} \]  

(29)
PID controller [for the transfer function of the controlled plant (2)]

\[ T_i^* = 2T_i - T, \quad T_o^* = 0.25T_i^*, \quad k_p^* = \frac{T_i^*}{k_i(1.28T + 2.72T_{d1})} \]  \( (30) \)

In relations (29) and (30) for the controller gain \( k_p^* \) the numeric constants are rounded to two decimal places, the approximate equalities are substituted by accurate equalities and the adjustable parameters are marked with a star (*) to make it clear that the forms of these formulas are final.

If the controlled plant is described by gain \( k_i \), the times \( t_{0.33} \) and \( t_{0.7} \) [see relations (3) and (4)], then relations (29) and (30) can be expressed in the forms appropriate for the direct computation of the values of the adjustable parameters of the controllers:

**PI controller**

\[ T_i^* = 1.25(t_{0.7} - t_{0.33}) - 0.5T, \quad k_p^* = \frac{T_i^*}{k_i(1.28T + 4.07t_{0.33} - 1.35t_{0.7})} \]  \( (31) \)

**PID controller**

\[ T_i^* = 1.59(t_{0.7} - t_{0.33}) - T, \quad T_o^* = 0.25T_i^*, \quad k_p^* = \frac{T_i^*}{k_i(1.28T + 5.27t_{0.33} - 2.55t_{0.7})} \]  \( (32) \)

Also in these relations the numeric constants are rounded to two decimal places.

It is easy to show that relations (29) – (32) can be used for \( T' = 0 \) for the corresponding analog controllers (Vitečková 1996).

The relations (29) and (30) or (31) and (32) for the monotone self-regulating plants with a time delay make it possible to determine the values of the adjustable parameters of the digital and analog PI and PID controllers, which ensure the marginal monotone control process. If a faster step response is desired and the overshoot is admissible, then it is necessary to experimentally increase the gain until the overshoot of the step response has the desired admissible value.

**Example 2**

For the controlled plant with the transfer function (7) from Example 1 it is necessary to design digital and analog PI and PID controllers and to tune them so that the control processes are of the marginal monotone type.

**Solution:**

Use the results from Example 1. For transfer function (8), the parameters \( T_1 = 3.48, \ T_{d1} = 7.96, \ k_i = 1 \) are valid. Likewise, for transfer function (9), \( T_2 = 2.22, \ T_{d2} = 6.73, \ k_i = 1 \).

**Digital PI controller** (\( T > 0 \))

The sampling period is chosen \( T = 1 \) s. From relations (29) for \( T = 1 \) we get:

\[ T_i^* = T_1 - 0.5T = 2.98, \quad k_p^* = \frac{T_i^*}{k_i(1.28T + 2.72T_{d1})} \approx 0.13 \]

**Analog PI controller** (\( T = 0 \))

From relations (29) for \( T = 0 \), it is:

\[ T_i^* = T_1 = 3.48, \quad k_p^* = \frac{T_i^*}{2.72T_{d1}k_i} \approx 0.16 \]

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Digital PID controller (T > 0)
The sampling period is chosen T = 1 s. From relations (30) for T = 1 we have:

\[ T'_1 = 2T_2 - T = 3.44, \quad T'_0 = 0.25T'_1 = 0.86, \quad k'_p = \frac{T'_i}{k_i(1.28T'_i + 2.72T'_d)} \approx 0.18 \]

Analog PID controller (T = 0)
From relations (30) for T = 0 is obtained:

\[ T'_1 = 2T_2 = 4.44, \quad T'_0 = 0.25T'_1 = 1.11, \quad k'_p = \frac{T'_i}{2.72T'_d k_i} \approx 0.24 \]

The obtained control system step responses are shown in Fig. 2.

![Fig. 1 Plant and approximations step responses](image1.png)

![Fig. 2 Control system step responses](image2.png)

4 Conclusions
The described PI and PID controller tuning method for monotone self-regulating plants with a time delay is simple and effective. Using the analytic way the values of the adjustable parameters of the controllers were obtained, which ensures the marginal monotone control process. A fast response with the desired overshoot can be obtained experimentally by “tuning in” only the controller gain.

References


**Reviewer:** doc. Ing. Zora Jančíková, CSc.