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HOT-AIR AGGREGATE CONTROL DESIGN

NÁVRH ŘÍZENÍ PRO TEPLOVZDUŠNÝ AGREGÁT

Abstract

The paper deals with robust control design for nonlinear control systems. The described robust control algorithms are based on state variables aggregation method, which is suitable for synthesis of relatively wide class nonlinear system. The described control algorithms were applied to flow air control of the hot-air aggregate, which were made as a laboratory models on Department of Automatic Control and Instrumentation. The designed control algorithms were verified by using computer simulations and also directly on laboratory model. The results achieved confirm suitable technical means and synthesis for nonlinear control tasks.

Abstrakt


1 INTRODUCTION

The goal of synthesis is design a control, which ensures required state trajectory tracking by a real state trajectory and also minimizes the quadratic objective functional. We assume the functional has the presentation

\[ J = \int_0^\infty (q^T(s)q(s) + s^T T s) dt \tag{1} \]

\[ s = -De, \quad e = x^*-x, \tag{2} \]

\[ \lim_{t \to \infty} e(t) = \lim_{t \to \infty} \dot{e}(t) = 0, \quad \lim_{t \to \infty} s(t) = \lim_{t \to \infty} \dot{s}(t) = 0, \tag{3} \]

where:

- \(D\) – aggregation matrix with constant elements,
- \(T\) – matrix with time constants,
- \(e\) – error,
- \(s\) – aggregation error,
- \(Q\) – vector function dimension \(m\) satisfying the conditions

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\[ q(s) = -q(-s), \quad q(0) = 0, \quad (4) \]

\[ s^T q(s) > 0 \quad \text{for} \quad s \neq 0. \quad (5) \]

Equation (1) can be modified [4] and we obtained nonlinear differential equation

\[ \dot{s} + T^{-1}q(s) = 0, \quad s(0) = s_0 \quad (6) \]

with equilibrium state \( s = 0 \), which describes behaviour of closed-loop control systems with respect to the aggregation error. It can be rewrite with the respect to error

\[ De = -T^{-1}q(De). \quad (7) \]

The control algorithm is be obtained by solving (7). This control minimizes the quadratic functional, but it also has to be asymptotic stable. Using Ljapunov method can prove it: Nonlinear control system (6) is asymptotic stable only if the derivation of Ljapunov function

\[ V = \frac{1}{2} s^T s \quad (8) \]

is negative definitive for \( s \neq 0 \), that is

\[ \dot{V} = s^T \dot{s} < 0. \quad (9) \]

This condition is fulfilling, if the (4), (5) are satisfy [5].

Let the disturbances of controlled nonlinear system is measurable and uncertainties do not occur, then the closed-loop control can be obtained on the basis of nominal mathematical model and conditions (4), (5)

\[ u = -[DG(x,v,t)]^{-1} \left[ T^{-1}q(s) - D(\dot{x}^* - f(x,v,t)) \right], \quad (10) \]

which ensures required state trajectory tracking \( \{x^*(t)\} \) with minimum of the quadratic objective functional (1). The closed-loop control properties depend very strongly on the suitable choice of vector function \( q \). There are many possibilities, for example,

\[ q(s) = s, \quad (11) \]

in this case, closed-loop control systems is linear, aggregation error course to the equilibrium state is exponential according to the time constant matrix \( T \). The control algorithm is without any limit. The vector function can have presentation

\[ q(s) = \text{sgn}(s), \quad (12) \]

where:

\[ \text{sgn} \quad \text{signum function}, \]

\[ \text{sgn}(s) = [\text{sgn}(s_1), \text{sgn}(s_2), \ldots, \text{sgn}(s_n)]^T, \]

\[ \text{sgn}(s_j) = \begin{cases} 1 & \text{for} \quad s_j > 0 \\ -1 & \text{for} \quad s_j < 0 \end{cases}. \]

The control is non-continuous, with high robustness and also high activity (quick switching between marginal values). This can be removed, when the saturation function, as a continuous substitution, is used.

\[ q(s) = \text{sat}(\Theta s), \quad (13) \]
where:

\[ \text{sat} \quad - \text{saturation function}, \]

\[ \text{sat}(\Theta s) = [\text{sat}(\Theta_1 s_1), \text{sat}(\Theta_2 s_2), \ldots, \text{sat}(\Theta_m s_m)]^T, \]

\[ \text{sat}(\Theta_1 s_1) = \begin{cases} \text{sgn}(\Theta_1 s_1) & \text{for } |\Theta_1 s_1| \geq 1, \\ \Theta_1 s_1 & \text{for } |\Theta_1 s_1| < 1. \end{cases} \]

\[ \Theta = \text{diag}([\Theta_1, \Theta_2, \ldots, \Theta_m]), \Theta_j \geq 1. \]

The high activity can be also removed, when the hyperbolic tangent function, as a smooth continuous substitution, is used.

\[ q(s) = \text{tgh}(\Theta s), \quad (14) \]

where:

\[ \text{tgh} \quad - \text{hyperbolic tangent function}, \]

\[ \text{tgh}(\Theta s) = [\text{tgh}(\Theta_1 s_1), \text{tgh}(\Theta_2 s_2), \ldots, \text{tgh}(\Theta_m s_m)]^T. \]

2 THE HOT-AIR AGGREGATE

The robust controls were applied to the hot-air aggregate, which is a laboratory model designed on Department of Control Systems and Instrumentation. The scheme of laboratory model is shown on Fig. 1, we can see the block schema of experimental laboratory model as a physical model of air-conditioning. It consists of a lamp (hot source) and a fan (flow air source) located in a tunnel and fed by a controlled supply voltage. In tunnel there are also many sensors for measuring the temperature and flow air. When we consider the model as SISO systems, we have two control tasks: 1. a temperature control of aggregate, as a disturbance it is possible to introduce the flow air, 2. a flow air control. The other possibility is that the model is considered as MIMO system, where we can control both variables, temperature and flow air [3].

![Fig. 1 The scheme of laboratory model](image-url)
2.1 Design of robust controls

The mathematical model was obtained by experimental identification from measured step responses. We found out that the hot–air aggregate can be written by two second-order differential equations, and mathematical model in state space has presentation

\[
\begin{align*}
\dot{x}_1 &= x_1, & x_{10} = x_1(0), \\
\dot{x}_2 &= -\frac{T + T_2}{TT_2} x_2 - \frac{1}{TT_2} x_1 + \frac{k}{TT_2} u, & x_{20} = x_2(0), \\
y &= x_1,
\end{align*}
\]  

(15)

where:

- \(x_1\) - flow air,
- \(T_i\) - time constants,
- \(u\) - control (supply voltage of fan).

That is why the aggregation matrix and matrix of time constant for each control variable are presented as

\[
D = d' = \begin{bmatrix} 1 & T_w \\ \frac{1}{T_w} & 1 \end{bmatrix}, \quad T = T_w, \tag{16}
\]

where:

- \(T_w\) - constants chosen with the respect for required course of closed control circuit (marginal a-periodical course).

The aggregation error has presentation

\[
s = -d'e = -\frac{1}{T_w} e_i - e_i, \tag{17}
\]

where:

- \(e_i\) - difference between required and real temperature,
- \(e_i = \dot{e}_i\).

According to the (10) the closed-loop control has presentation

\[
u = -\frac{TT_2}{kT_w} \left[ \frac{1}{T_w} q(s) - \frac{1}{T_w} e_i - e_i - \frac{T + T_2}{TT_2} x_2 - \frac{1}{TT_2} x_1 \right]. \tag{18}
\]

By choosing \(q(s)\) the following control algorithms are obtained:

\[
q(s) = s, \quad u = \frac{TT_2}{kT_w} e_i + \frac{TT_2}{k} \left( \frac{1}{T_w} + \frac{1}{kT_w} \right) e_i + \frac{T + T_2}{kT_w} x_2 + \frac{1}{k} x_1,
\]

\[
q(s) = \text{sgn}(s), \quad u = \frac{TT_2}{kT_w} \text{sgn} \left( \frac{1}{T_w} e_i + e_i \right) + \frac{TT_2}{kT_w} e_i + \frac{T + T_2}{kT_w} x_2 + \frac{1}{k} x_1,
\]

\[
q(s) = \text{sat}(\delta s), \quad u = \frac{TT_2}{kT_w} \text{sat} \left( \frac{1}{T_w} e_i + e_i \right) + \frac{TT_2}{kT_w} e_i + \frac{T + T_2}{kT_w} x_2 + \frac{1}{k} x_1,
\]

\[
q(s) = \text{tgh}(\delta s), \quad u = \frac{TT_2}{kT_w} \text{tgh} \left( \frac{1}{T_w} e_i + e_i \right) + \frac{TT_2}{kT_w} e_i + \frac{T + T_2}{kT_w} x_2 + \frac{1}{k} x_1.
\]
2.2 Verification of designed control algorithms

The designed control algorithms were verified by a computer simulation with program MATLAB – SIMULINK and directly on the real laboratory model of hot-air aggregate – see Fig. 2 – 5. The laboratory model is connected to PC with help of data measured card AD512; the control algorithms were realized in program MATLAB.

Fig. 2 Experimental results for control with $q(s) = s$, the course of flow air

Fig. 3 Experimental results for control with $q(s) = s$, the course of control

Fig. 4 Experimental results for control with $q(s) = \tgh(\delta s)$, the course of flow air

Fig. 5 Experimental results for control with $q(s) = \tgh(\delta s)$, the course of control
3 CONCLUSIONS

The contribution presents nonlinear control systems synthesis, which is solved by using robust controls. There are described properties of three types of control algorithms: linear, discontinuous and continuous. The described types of control were used for flow air control of hot-air aggregate. Generally, all robust control algorithms ensured reaching the required value.

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REFERENCES


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