This paper addresses problems of bankruptcy prediction. It has been oriented by past and recent situations in the Czech Republic. A brief overview of authors from abroad, who have been involved in investigations of this subject, is given. The circulation of capital is described inclusive intrusive factors that disturb capital’s circulation hampering economic functioning of corporate businesses, which may lead to their final demise. A set of indicators has been specified that reflect the circulation of capital, and which provide for employing a statistical process – discriminate analysis – that links the probability of a corporate business default to these indicators.

Key words: Bankruptcy, capital circulation, discriminate analysis

INTRODUCTION

Metallurgy, mining industries, heavy machine engineering, and many other industries are characterised by high investment cost. It is essential for any investor to be able to assess the current financial situation, and to predict future developments of companies in which they are going to invest. It is a key information whether a company is exposed to danger of bankruptcy, as default concerns not only the firm itself or its potential investors but affects also all businesses that cooperate directly or indirectly with the firm. In the case that a corporation is a major employer in the region, the default may have serious consequences for the whole regional community life and culture. Bankruptcy means reduced values of financial portfolios, loss of jobs, loss of individual revenue, loss of tax revenues for governing bodies, etc. As such it is not surprising that economic intelligence with its insolvency prediction is gaining an ever increasing importance. The default is usually an outcome of development whose negative trends were active over a number of past years, and which were reflected by idiosyncrasies of capital structure, as well as modes of actual production. In a perfect world, there would be a tool that would enable instant and completely reliable bankruptcy predictions. This is not that case, as yet, but a lot has been already made.

DEFAULT PREDICTIONS

Initial trials of solving the problem can be traced back to works that employed pair criteria analysis by comparing financial ratios of successful companies with those that failed. The works of Ramser and Foster [1], Fitzpatrick [2], Winaker and Smith [3], belong all here. Beaver [4] analysed such financial ratios, and Altman [5] acknowledges reference. E. I. Altman is the dean of default predictors. He was the first person who successfully used statistical techniques of discriminate analysis that were widely followed as reported by Altman et al. [6]. Another group of researchers tried to improve the discriminate analysis by introducing more sophisticated sets of variables – Norton and Smith [7] or by stabilising ratio indicators – Dambolina and Khoury [8]. Tam and Kiang [9] used methods of neural networks to realise models of insolvency prediction. The most recent investigations have employed cash-flows – Henebry [10], or financial market parameters – Curry et al. [11]. Other examples are given by Hol et al. [12]. Bankruptcy prediction models have been used for predicting failure of firms operating abroad – for example Altman et al.
Also works on prediction methodology have been published – Pindado and Rodrigues [14]. Other references could be given also. Nevertheless there is currently no agreement in regard of the priority of individual factors used for default prediction purpose.

CORPORATE BANKRUPTCY FROM THE PERSPECTIVE OF CAPITAL CIRCULATION

Concerning the assessments of a company economic situation, the choice of assessment variables should use some theoretical framework for its starting point. As already stated, bankruptcy stays at the end of longer negative developments in course of which the economic results have changed the capital structures. This has been observed taking into account economic developments of mining enterprises in the Czech Republic. Many of these were closed for economic reasons – Dvořáček et al. [15]. A capital circulation perspective can provide for a starting theoretical framework.

From the economic point of view, any enterprise activity can be regarded as a circulation of capital. Originally, it is the capitalist’s own capital (share capital, etc.) or the outside capital (credits, loans, etc.). The capital pays for fixed assets (landed property, buildings, machines, etc.), current assets (raw materials and others inputs), and hiring of workforce. A classical production process starts with the processing of raw materials or semi-products which operation adds the value of fixed assets (depreciations) and workforce to the semi-finished products. The semi-finished products’ further processing is finalised by the production of goods – wares for the market. Marketing and sales of the wares changes them into receivables that – being realised – transform again to capital. This capital is a fresh input of the running process of the capital circulation. It refreshes consumed current assets, pays for the workforce, and redresses the balance of fixed assets. Capital resources can be also expanded by interventions from outside so that the process of assets’ rejuvenation may accelerate on a higher level.

The refreshment of current assets and paying the working force must be continuous; otherwise reproduction reductions follows – both fixed assets and employment force decrease, which processes reach soon their limits. Capital circulation disruption means diminished ‘returns’ – there is less capital at the beginning of the reproductive circle, and a necessity of outside intervention by providing an extra capital of one’s own or from other independent capital resources. This in fact can be done but only for limited periods of time.

Capital disruptions may arise in all its phases of circulation. Among these are:

- Difficulty or impracticability of equity capital extensions (owner’s investment, etc.),
- Difficulty or impracticability of getting debt capital (bank loans, liabilities),
- Problematic maintenance of operational cash flows as results of: (i) regarding input (for example reduced purchases of raw materials due to lack of capital), (ii) regarding output (problematic sales of produced goods). This results in in-the-red operations (which make the problem of disrupted cash flow even worse),
- Problematic or impossible sales of fixed assets.

Consequently, fixed and current assets, and equity decreases are inevitable outcomes of these disruptions. Outside capital intervention is rather difficult to predict, as several factors are involved:

- Employment restrictions, and input component restrains confine liabilities,
- Capital scarcity delays invoice payments, which in its turn increase liabilities,
- Bank loans become stagnant because difficult businesses cannot get them, and they do not pay instalments for current loans. Sometimes they pay back only to evade creditor induced bankruptcy.

All this provide for a basis on which the following indicators of business economic footing can be defined: business production activities are reflected by ratio indicators ‘Receivables/Current Assets’, and ‘Reserves/Current Assets’. Economic outcome developments have their expression in ‘Equity Capital/Total Assets’. Nevertheless business economic developments occur in time, and it is necessary to define an indicator that would take this fact into account. Capital structure temporal indicators (indices) are: Fixed Assets Index, Current Assets Index, Receivables Index, Past Income Index, and Equity Index. Evaluations of business economic footing can be made by statistical methods of discriminate analysis.

APPLYING DISCRIMINATE ANALYSIS

The discriminate analysis enables evaluations of differences between two or among several groups of multiple feature objects. It is usually classified as techniques that either interpret differences between a priori groups of objects or those that aim at structuring objects into classes. Attributes of an object are compared with attributes of other objects.

The classical discriminate analysis statistically investigates relations between a group of independent attributes, \( p \) – discriminators, and a single qualitatively dependant variable – output, \( G \). The simplest case is represented by a binary variable output of 0 (zero) value if an object belongs to Class I. If an object belongs to Class II, the binary variable output has a value of 1.

Classes are clearly distinct and each object belongs to one of them. The objective is to develop a prediction model that would provide for structuring of new objects to classes. Another purpose may also be the identification of such attributes that contribute to processes of classification.

We shall solely take into account a standard probability approach assuming all group attributes to be normally distributed with differences concerning only
some of normal distribution parameters. The symbol, \( \pi_j \), designates an apriori probability of an object to belong to group I. The equation, \( \pi_j = 1 - \pi_i \), expresses probability of an object to belong to another group.

The Bayes’s theorem provides for defining of an object’s posterior probability of belonging to \( j \)-group \( (j = 1, 2) \)

\[
P(G = j \mid x) = \frac{f_j(x)}{\sum_{i=1}^{2} f_i(x) \pi_i}, \quad j = 1, 2, \tag{1}
\]

where

\[
f_j(x)\text{is probability density of primary class, } j,
\]

\[
x^r=(x_1, x_2, ..., x_n) \text{ is discriminator value vector.}
\]

If \( P(G = j \mid x) \) is known, a decision-making rule can be applied which enables structuring of new objects in classes with higher posterior probability. It is obvious that false classification may occur. It can be demonstrated that such error occurrence is minimal if the posterior probability, \( P(G = j \mid x) \), has been chosen as a decision-making rule.

The equation (1) makes for classifying objects to class I if \( \pi_j f_j(x) > \pi_i f_i(x) \) because the denominator sum total is just a standardising constant.

The concrete discriminate rule application will depend on parameter differences of both distributions. It is a case of linear discriminate analysis if \( f_j(x) \), and \( f_i(x) \) are only in expected value different. If the distribution also differs in covariance matrices, then the quadratic discriminate analysis should be applied.

### A practical application of the linear discriminate function for two classes

We start with known matrices, \( X_1 \), of size, \( n_1 \times m \), for class I, \( X_2 \), and size, \( n_2 \times m \), for class II. Individual objects in matrix, \( X \), of all data are classified along the output, G. Sample means, \( \bar{x}_1 \), \( \bar{x}_2 \), are numerically expressed for all classes, and a common covariance matrix, \( S = \frac{(n_1 - 1)S_1 + (n_2 - 1)S_2}{n_1 + n_2 - 2} \),

where \( S_1 \), \( S_2 \) are the covariance matrices of individual classes.

An appropriate method will provide for a priori probability, the simplest assumption is: \( \pi_1 = \pi_2 = 0.5 \).

If a single-piece choice is made that is subsequently structured into classes, relative frequencies can be applied:

\[
\hat{\pi}_1 = \frac{n_1}{n_1 + n_2}, \quad \hat{\pi}_2 = \frac{n_2}{n_1 + n_2}.
\]

Assuming normality, linear discriminate function coefficients can be specified from estimations

\[
\hat{a} = (\bar{x}_1 - \bar{x}_2)S
\]

and

\[
\hat{b} = -\frac{1}{2} \hat{a}^r(\bar{x}_1 - \bar{x}_2) - \ln \left( \frac{\hat{\pi}_1}{\hat{\pi}_2} \right).
\]

Classifying new objects with attribute values, \( x_0 \), such rule is applied that puts an object in Class I if \( \hat{a}^r x_0 + \hat{b} > 0 \).

Inversely, the object is put in Class II.

The linear discriminate analysis was applied to 73 enterprises that were structured in two classes of sound (40) and failed (33) businesses. To each business a set of discriminators (as defined in the preceding chapter) was attributed. Bankruptcy announcement timing is designated, (0); concrete values for computing are given by Balance Sheet as to the date, 31, 12, of the year, (t-1), preceding the bankruptcy date. The same values are established for the year, (t-2). Ratio indices were calculated from (t-1) values so that they preceded bankruptcy for maximum of 12 months. The average period between the Annual Balance Sheet issue and bankruptcy announcement was of 6.8 months. The development indices were established by relating (t-1) values to values of the period, (t-2).

The objective is to find a discriminate function that would provide for classifying of sound and failed enterprises as characterised by the defined discriminators.

Class I (40 sound businesses) leads to expected values

\[
(0.57, 0.20, 0.51, 1.15, 1.09, 1.04, 232.53, 1.16)
\]

and a covariance matrix

\[
\begin{bmatrix}
0.04 & -0.01 & 0.02 \\
-0.02 & 0.00 & 0.04 \\
0.01 & 0.02 & -0.01 & 0.43 \\
-0.01 & 0.00 & -0.01 & -0.03 & 0.06 \\
0.01 & -0.01 & 0.02 & -0.06 & 0.04 & 0.11 \\
46.58 & -253.7 & 29.17 & -250.4 & 28.56 & 76.58 & 2139876.70 \\
-0.01 & 0.00 & -0.02 & 0.01 & 0.02 & 0.03 & -3189.08
\end{bmatrix}
\]

Class II (33 defaults) lead to expected values

\[
(0.65, 0.19, -4.47, 0.86, 0.73, 0.77, 10.64, -1.13)
\]

and a covariance matrix

\[
\begin{bmatrix}
0.11 & -0.03 & 0.06 \\
1.38 & 0.63 & 449.23 \\
0.02 & -0.02 & 2.61 & 0.86 \\
-0.01 & 0.02 & 2.72 & -0.02 & 0.36 \\
0.06 & 0.01 & 313 & -0.05 & 0.22 & 0.40 \\
2.07 & -217 & 4994 & 0.93 & -0.42 & 18.85 & 3471.45 \\
-0.25 & -0.01 & 312 & -0.61 & 0.36 & 0.45 & 22.43 & 817
\end{bmatrix}
\]

The discriminate function coefficients, \( a_1, ..., a_8 \), are calculated:

\[
a = S^{-1}(\bar{x}_1 - \bar{x}_2) = (-0.07, 0.37, 0.01, 0.78, 1.27, 0.03, 0.00, 0.61)
\]

If expected value vectors of Classes I and II are substituted in the equation; we can calculate average values:

\[
\bar{Z}_1 = 3.096, \quad \bar{Z}_2 = 0.936.
\]
The optimal threshold value, $C$, which determines objects for class I or II, can be calculated:

$$C = \frac{Z_1 + Z_2}{2} = 2,016.$$  

Enterprises whose linear discriminate function values are in excess of 2,016 can be classified as sound businesses. Linear discriminate function values under 2,016 imply default classification.

If the computed linear discriminate function is applied to classification of our set of enterprises, then classification for sound enterprises is 100% successful. For default enterprises, the prediction success rate is 70%.

**DISCUSSION**

From the point of view of practical applicability, it is the degree of correspondence between the predicted and actual state or development that is the measure of success of the prediction method used. If comparisons are made between the work of Altman, (although his method of data provision is not identical), we can conclude:

**Altman** [16]:
- File: 33 default firms
- 33 sound firms

**Input data:**
- 1 year before default in one case
- 2 years before default in another case

**Prediction reliability from values of the year preceding default:**
- Correctly classified: 94% of default firms
- Correctly classified: 97% of sound firms

**Prediction reliability from values of two years preceding default:**
- Correctly classified: 72% of default firms
- Correctly classified: 94% of sound firms

**Our results:**
- File: 33 default firms
- 40 sound firms

**Input data:**
- 1 to 2 years before default

**Predictability rate:**
- Correctly classified: 70% of default firms
- Correctly classified: 100% of sound firms

Rates of success are comparable, taking especially into account the fact that three variables were determined within one-year default preceding periods, and five variables within two years before defaults.

**CONCLUSION**

Our bankruptcy prediction modelling has been difficult because default of some firms was not the outcome of their bad finances but rather was the result of creditors’ speculative pressure. Taking this condition into account, a widening of data files, and looking for other sources of their provision can make for further development of this prediction method.

**REFERENCES**


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