Sborník vědeckých prací Vysoké školy báňské - Technické univerzity Ostrava
číslo 2, rok 2007, ročník LIII, řada strojní
článek č. 1550

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THE POSITIONAL DEVIATION IN TWO AXES

PRESNOSŤ POLOHOVANIA V DVOCH OSIACH

Abstract

The positional deviation (difference between the actual and target position) belongs to the important criteria that describe the performance of numerically controlled axes. The procedure for determination of such deviation is described in the international standard ISO 230-2:1997. This standard provides calculation of the positional deviation only in several discrete (measuring) points. Moreover it does not consider effects of the measuring instrument on the obtained results. The new methodology is adopted and it is enables estimation of the positional deviation in any point of the axis travel, together with the uncertainty of such estimate. Obtained results can be incorporated into a control system in the form of corrections enhancing positioning possibilities of individual axes.

The paper introduces procedures that were verified by measurements for one linear axis. The more complicated situation occurs for testing the positioning accuracy in a plane or in a space respectively. Therefore possible solutions for determination the repeated positioning accuracy in any point of the plane are presented at the end of the paper, together with expression of the respective uncertainty.

Abstrakt

Odchýlka polohovania (rozdiel medzi skutočnou a požadovanou polohou) patria medzi významné kritéria opisujúce činnosť počítačovo riadených osí strojov. Postup na určenie takejto odchýlky sa uvádza v medzinárodnej norme ISO 230-2:1997. Táto norma poskytuje návod na výpočet odchýlky polohovania iba v niekoľkých diskrétnych bodoch (bodoch merania). Okrem toho neuvažuje vplyv meracieho zariadenia na získané výsledky. Preto sa navrhuje nová metodika, ktorá umožňuje odhad odchýlky polohovania v ľubovoľnom bode na osi, spolu s neistotou takéhoto odhadu. Získané výsledky sa dajú zahrnúť do riadiaceho systému vo forme korekcií umožňujúcich zlepšenie schopnosti polohovania jednotlivých osí.

V článku sa uvádzajú postupy, ktoré boli meraniami overené pre jednu lineárnu os. Zložitejšia situácia platí pri testovaní presnosti polohovania v rovine, resp. v priestore. Preto sú na záver uvedené úvahy o možných postupoch pri vyjadrení opakovanej polohovateľnosti aj s neistotami v ľubovoľnom bode roviny.

1 INTRODUCTION

Testing of the positional deviation of the numerically controlled axis (either rotary or longitudinal) is ruled by the international standard ISO 230-2:1997 [1]. This standard provides guide for design of the test, testing conditions and also evaluation procedure for processing the measured data. In general, the testing procedure is based on repeated measurements of the actual position of the tested axis in several discrete points (target positions), located equally along the axis travel.

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The several parameters dealing with positional deviation can be measured and calculated according to such a scheme. The evaluation of measured data according to the standard gives just estimation of the device performance in several discrete points (measurement points P_i). But the course of individual parameters among the measurement points is just roughly estimated according to the standard, giving no warranty on correctness of the results in between points.

2 EVALUATION OF MEASURED DATA ACCORDING TO THE STANDARD

The above mentioned standard introduces evaluation of the measured data that is aimed namely at determining the maximum positional deviation over the whole axis (measurement) travel. The evaluation of results covers calculation of the parameters related to the positional deviation in each of the measurement points P_i, covered also by the deviation boundaries $\bar{x}_i\uparrow +3s_i\uparrow$; $\bar{x}_i\uparrow - 2s_i\uparrow$ (respectively $\bar{x}_i\downarrow +2s_i\downarrow$; $\bar{x}_i\downarrow -3s_i\downarrow$ in reversal direction).

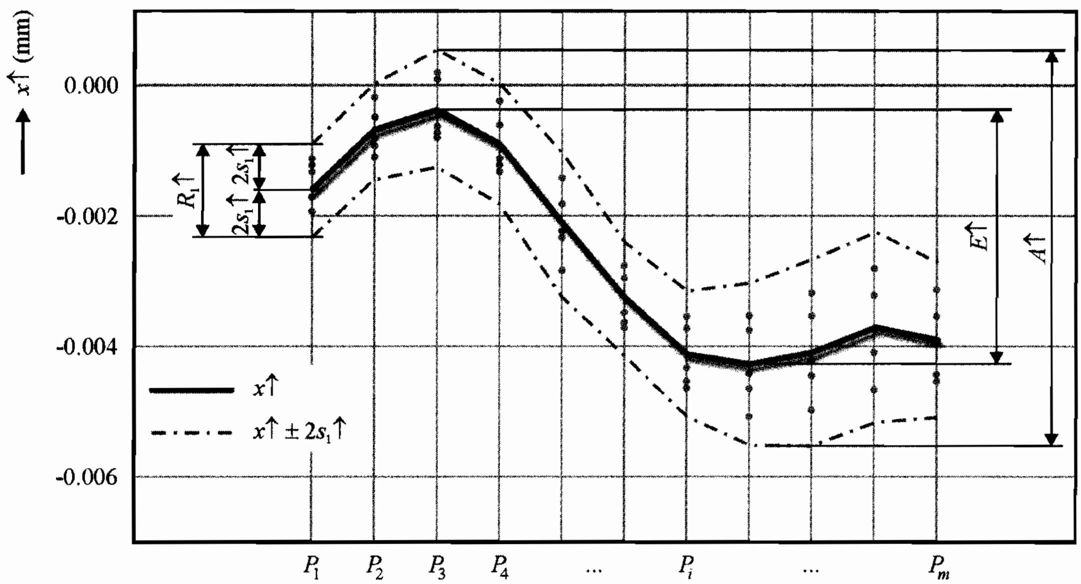

Fig. 1 Example of the graphical presentation of evaluated positional deviation according to the standard [1] (results plotted for unidirectional positioning)

Note that the deviation boundaries are calculated only from measured data, considering their variation as caused only by a tested machine and not influenced by the measurement instrument itself. This is valid only when the accuracy of measurement instrument is much better than expected positional deviations. But for modern numerically controlled axes, operating with resolutions of 0.001 mm, this is not always the case and the used measuring instrument itself can significantly affect the measurement results [2].

As you can observe from Figure 1, the standard assumes linear course of the positional deviation among the measurement (testing point), as well as linear course of the deviation boundaries.

3 EVALUATION OF THE POSITIONAL DEVIATION IN ANY POINT OF THE AXIS TRAVEL

When considering the above presented measurement scheme, following positional deviations are measured in individual measurement points [3]:

$$P_1:\ \Delta_{11}, \Delta_{12}, ..., \Delta_{1j}, ..., \Delta_{1n}$$

$$P_2:\ \Delta_{21}, \Delta_{22}, \ldots, \Delta_{2j}, \ldots, \Delta_{2n} \tag{1}$$

$$\vdots$$

$$P_m:\ \Delta_{m1}, \Delta_{m2}, \ldots, \Delta_{mj}, \ldots, \Delta_{mn}$$

where

i – to m is the running number of the measurement point ,

Δ_{ij} – are positional deviations,

j – 1 to n is the running number of the measurement of positional deviation in a given measurement point. It is assumed that the same number n of measurements of the positional deviation is performed in each measurement point.

(*Remark: the positional deviation that is designated in the standard [1] as x_{ij}, is for sake of better clarity and understandability designated by different symbol Δ_{ij}).

If we want to obtain the estimates of the positional deviations also in other points than the measurement ones, we must approximate course of estimates. The least squares method is suitable for such approximation. The curve in a form of polynomial of the third order will be placed over the points $(P_1, \overline{\Delta_1})$, $(P_2, \overline{\Delta_2})$, ..., $(P_i, \overline{\Delta_i})$, ..., $(P_m, \overline{\Delta_m})$:

$$\Delta = a + b \cdot P + c \cdot P^2 + d \cdot P^3 \tag{2}$$

where

Δ – is the positional deviation of the target position and actual position in any point P and $P \in \langle P_1; P_m \rangle$,

a, b, c, d – are unknown parameters of the polynomial.

Besides that we want to determine also expanded uncertainty U of estimate of the positional deviation Δ in any point P. To be able to do this, we must determine the estimates of polynomial parameters a, b, c and d, their uncertainties and covariances among them. Thus we leave the evaluation according to the Figure 1 and we get evaluation providing results according to the Figure 2. To do so, we need to introduce a completely new approach to evaluation of the measured data.

The positional deviation for each measurement j = 1 to n in each particular point P_i, with i = 1 to m, can be calculated as the difference between the target position and the measured actual position:

$$\Delta_{ij} = P'_{ij} - P_i \tag{3}$$

where

P_i – is the target (programmed) position,

P'_{ij} – is the actual (measured) position.

The actual (measured) position P'_{ij} comprises two components:

$$P'_{ij} = P_{ij} + \delta_{mer.} \tag{4}$$

where

P_{ij} – is the position indicated by the measuring instrument,

$\delta_{mer.}$ – is the measurement error in the particular point, in our case estimated as the maximum permissible error of the measuring instrument [7]. This means that the error remains constant in any point.

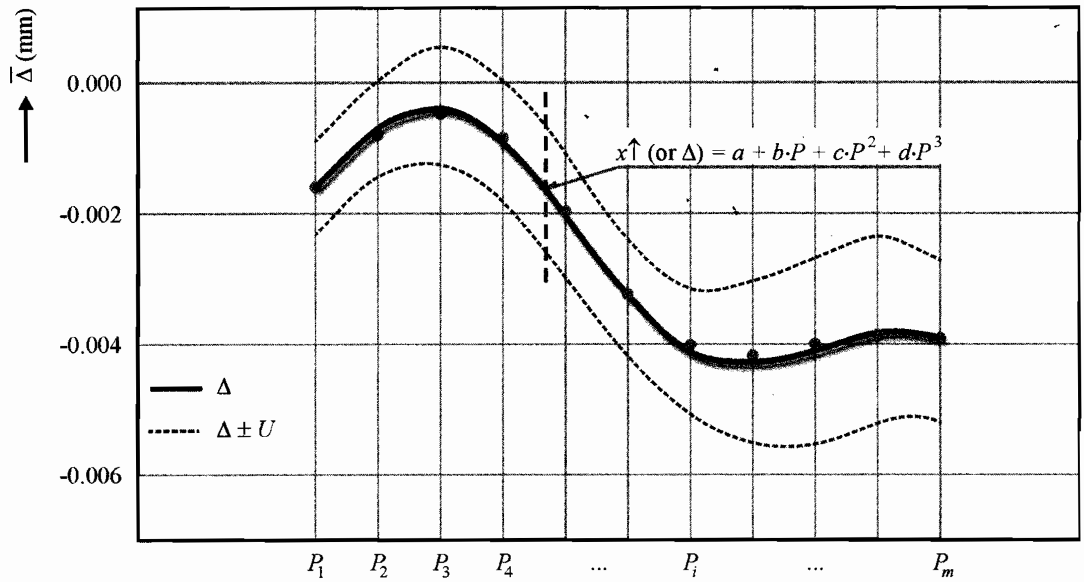

Fig. 2 Estimation of the positional deviation in any point of the axis travel

According to the previous calculations of estimates of the positional deviation (as the arithmetic means), the set of equations for j = 1 to n positional deviations in m points (target positions) can be expressed as [8]:

$$\begin{aligned} \overline{\Delta}_1 &= \overline{P}_1 - P_1 + \delta_{mer} \\ \overline{\Delta}_2 &= \overline{P}_2 - P_2 + \delta_{mer} \\ &\vdots \\ \overline{\Delta}_m &= \overline{P}_m - P_m + \delta_{mer} \end{aligned} \qquad (5)$$

where

$\overline{P}_i$ – is the estimate (obtained as an arithmetic mean) of the actual (measured) positions in any given point P_i.

The set of equations describing the positional deviations (5) can be written in a matrix form:

$$\boldsymbol{x} = \overline{\boldsymbol{P}} - \boldsymbol{i}\,\boldsymbol{P} + \boldsymbol{i}\,\delta_{mer} \qquad (6)$$

where

$\boldsymbol{x}$ – is the vector of the estimates of individual positional deviations (dimension m),

$\overline{\boldsymbol{P}}$ – is the vector of the estimates of measured actual positions (dimension m),

$\boldsymbol{P}$ – is the vector of target positions (dimension m),

$\boldsymbol{i}$ – is the unit vector (dimension m) and

$\boldsymbol{x} = \left(\overline{\Delta}_1, \overline{\Delta}_2, \cdots, \overline{\Delta}_m\right)^{\mathrm{T}}$

$\overline{\boldsymbol{P}} = (\overline{P}_1, \overline{P}_2, ..., \overline{P}_m)^{\mathrm{T}}$

$\boldsymbol{P} = (P_1, P_2, ..., P_m)^{\mathrm{T}}$

When taking the matrix notation into account, the covariance matrix $\boldsymbol{U}(\boldsymbol{x})$ can be written in a form (assuming $\boldsymbol{P}$ as non-random vector, $\overline{\boldsymbol{P}}$ and δ are independent):

$$U(\mathbf{x}) = U(\overline{\boldsymbol{P}}) + U(\boldsymbol{P}) + u^2(\delta)\boldsymbol{i}\,\boldsymbol{i}^T \tag{7}$$

The uncertainty of the target position is zero in our case (no influence on value of the target position) so that the covariance matrix $U(x)$ gets the form [4]:

$$U(\boldsymbol{x}) = U(\overline{\boldsymbol{P}}) + u^2(\delta)\boldsymbol{i}\,\boldsymbol{i}^T \tag{8}$$

After specifying the individual terms (because P_i a P_j are independent):

$$U(\boldsymbol{x}) = \begin{pmatrix} u^2(P_1) & 0 & \cdots & 0 \\ 0 & u^2(P_2) & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & u^2(P_m) \end{pmatrix} + u^2(\delta)\, i\, i^{\mathrm{T}} =$$

$$= \begin{pmatrix} u^2(P_1)+u^2(\delta) & u^2(\delta) & \cdots & u^2(\delta) \\ u^2(\delta) & u^2(P_2)+u^2(\delta) & \cdots & u^2(\delta) \\ \cdots & \cdots & \cdots & \cdots \\ u^2(\delta) & u^2(\delta) & \cdots & u^2(P_m)+u^2(\delta) \end{pmatrix} \tag{9}$$

The uncertainties $u(P_i)$, where $i = 1, 2, ..., m$, in the matrix (8) are evaluated by the type A method form measured data:

$$u(P_i) = \sqrt{\frac{1}{n(n-1)}\sum_{j=1}^{n}(P_{i,j} - \overline{P}_i)^2} \tag{10}$$

This procedure yields to the estimates of unknown parameters a, b, c, d, uncertainties of those estimates and covariances among them.

4 MEASUREMENTS

The Faculty of Mechanical Engineering, Slovak University of Technology in Bratislava, started to cooperate with the MicroStep, Ltd. that produces machines for industrial purposes. The CNC laser cutting machine with linear drives has been designed in cooperation with the former Department of production technique at the FME STU. The machine is of a gantry type with extreme dynamics of drives based on linear synchronous motors. The gantry is driven at both sides by direct linear motors provided by encoders for positional measurements. The PID controllers rule the performance of motors. The supporting part of gantry is equipped with a movable unit for adjusting the height of the technological head over the material being cut. The technological table is designed for work velocities reaching up to 260 m/min and acceleration up to 25 m/s^2. The axes lengths are as follows: 4670 mm for X axis, 2143 for Y axis and 317 mm for Z axis. The gantry can be positioned separately for each of the individual axes or the combination of movements can be executed as well. The positional deviation is measured by a laser interferometer as an external measuring instrument, providing results with lower uncertainty than the measuring system of the machine itself [5].

To verify the enhanced methodology of the repeated positioning accuracy, measurements in 10 points of the Y axis have been performed, doing 10 repeated measurements in each point (see Fig. 3, Fig. 4) [6]. The green curve shows evaluation according to the standard; the blue curve represents a regress function and shows the results of the proposed methodology. Regress curves for approaching from left and from right are expressed by following equations:

$$\Delta_{\text{zprava}} = 0.0072729 + 2.17\cdot10^{-5}\cdot P - 4.2836\cdot10^{-8}\cdot P^2 + 4{,}5325\cdot10^{-11}\cdot P^3 \tag{11}$$

$$\Delta_{\text{zľava}} = 0.0081716 + 2.49\cdot10^{-5}\cdot P - 4.7664\cdot10^{-8}\cdot P^2 + 4{,}6966\cdot10^{-11}\cdot P^3 \tag{12}$$

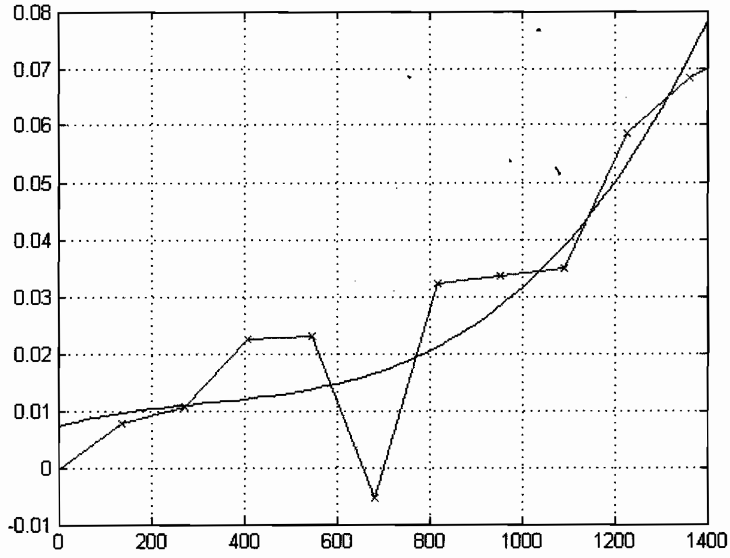

Fig.3 Approach from right

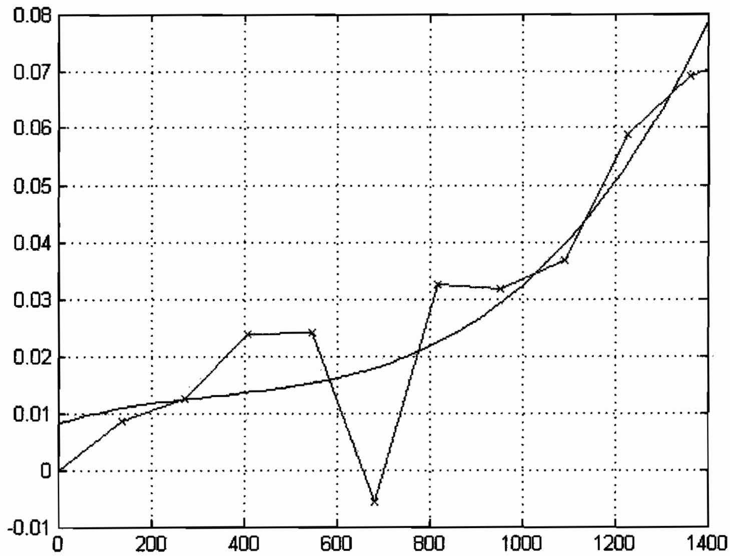

Fig.4 Approach from left

The regress curve characterises analytical expression of the correction curve. Therefore the desired (input) value can be corrected and the better positioning accuracy can be obtained. This means that corrected values are used as an input instead of the previously intended desired values.

5 MEASUREMENT MODEL IN THE TWO AXES

The basic model (2) without interactions for a two-dimensional case (see Fig. 5) gets the form

$$\Delta = a + b_1 \cdot P + c_1 \cdot P^2 + d_1 \cdot P^3 + b_2 \cdot R + c_2 \cdot R^2 + d_2 \cdot R^3 \qquad (13)$$

Then the expression (5) transforms to a form

$$\overline{\Delta}_{ij} = \overline{P}_{ij} - P_{ij} + \delta_{mer}$$

where index j represents the j-th coordinate R_j

and for $\boldsymbol{x(P,R)}$ that is now a function of coordinates P and R

$$\boldsymbol{x(P,R)} = \overline{\boldsymbol{P}}_j - \boldsymbol{i}\,\boldsymbol{P}_j + \boldsymbol{i}\,\delta_{mer} \qquad (14)$$

and if we introduce the above mentioned designation, we get

$$\overline{\boldsymbol{P}} = \begin{pmatrix} \overline{\boldsymbol{P}}_1 \\ \overline{\boldsymbol{P}}_2 \\ \vdots \\ \overline{\boldsymbol{P}}_m \end{pmatrix} \quad \text{a} \quad \boldsymbol{P} = \begin{pmatrix} \boldsymbol{P}_1 \\ \boldsymbol{P}_2 \\ \vdots \\ \boldsymbol{P}_m \end{pmatrix}$$

and the above mentioned procedure can be applied.

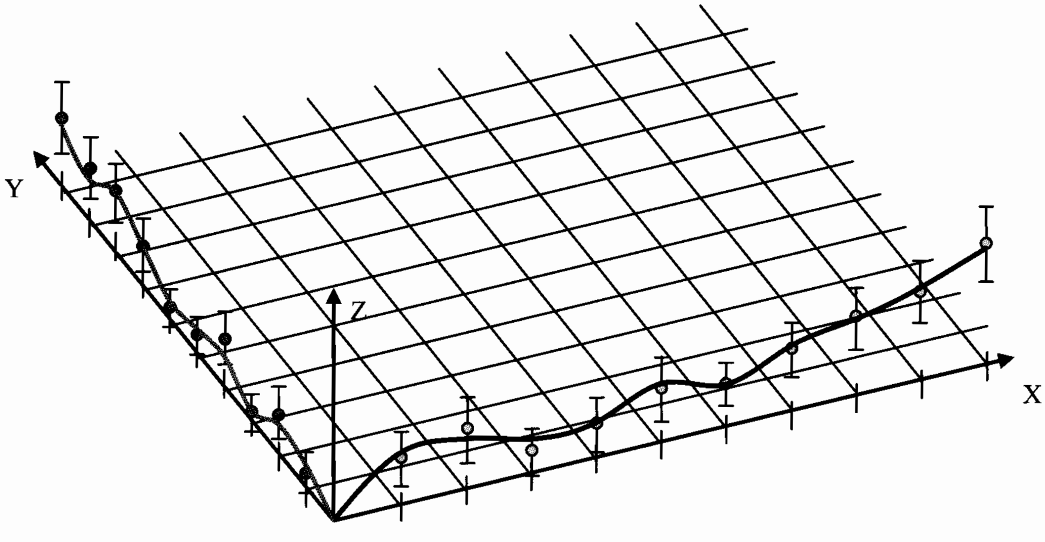

Fig.5 Measurements in two axes

6 CONCLUSIONS

The presented methodology gives the opportunity to estimate the positional deviation in any point of the axis travel, no matter whether rotational or longitudinal. Moreover it provides the estimate of the positional deviation with the respective uncertainty of such estimate. This gives the designer or programmer the possibility to build appropriate corrections into the control program or the adequate design corrections can be performed in the design of the machine.

The presented evaluation of measured data according to the standard shows similar behavior of the controlled axes when approaching to the desired position from both sides (positive and negative).

The best solution well be to estimate the positional deviation in any point of the surface. It means to create regression areas.

Acknowledgement

The research work described in the paper was performed by a financial support of the Slovak Scientific Grant Agency (VEGA), grant No. 1/3131/06.

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