ACTIVE VIBRATION REDUCTION OF RIGID ROTOR BY KINEMATIC EXCITATION OF BUSHES OF JOURNAL BEARINGS

Possibilities of active lateral vibration reduction of a symmetric, rigid rotor supported by journal bearings are given. They were obtained by computational modelling. Efficiency of the feedback P and PD controllers in the stable revolution interval was examined. The linearized rotor system model was used. The results of the theoretical analysis are assigned for a testing stand where the bearing bush motions are deactivated by piezoelectric actuators connected to the controllers.

Key words: bearing, controller, rotor system, vibration reduction


Ključne riječi: ležaj, upravljački sklop, sustav rotora, prigušivanje vibracije

INTRODUCTION

Centrifugal forces caused by the unbalance are the main source of vibration in rotating machines. The perfect balancing is very expensive, or it is not possible at all. Besides the unbalance distribution can vary in time, caused by abrasion or by support modifications. To suppress excessive vibration, higher damping is used first by passive tools such as absorbers and dampers, or by active tools are mostly used.

Nowadays, the use of magnetic bearings is proposed for the active control. In other cases, the use of different types of actuators such as pneumatic, hydraulic, electromagnetic [1, 2] was proposed for the active control. However, no one of them can show significantly better properties.

Recently piezoelectric actuators are used for active control frequently. Very promising results were achieved in the case of active control of a rotor supported by rolling bearings [3].

The aim of this work is to obtain the basic knowledge for the adjustment of the test stand designed to investigate the possibilities how the rotor vibration can be reduced by kinematic excitation of bearing bushes using the piezoelectric actuators. The actuators are located in horizontal and vertical directions where the direct kinematic excitation of the bushes is supposed (Figure 1).

Figure 1. Rotor system control layout

ANALYTICAL STUDY AND INSTABILITY LIMIT DETERMINATION

The investigated rotor system has the following properties: (i) the rotor is symmetric and rigid, (ii) the rotor is supported by two identical journal bearings and the oil-film forces are determined using the solution of Reynolds equation for cylindrical, short and by cavitation influenced bearing, (iii) a static unbalanced shaft is assumed, (iv) the centrifugal force caused by unbalance acts in the symmetry plane and (v) the rotor rotates at a constant angular velocity.

Under the given assumptions the motion equations of the rotor system lateral vibration in a stationary coordinate system x, y, z (Figure 2) can be written as:
After the linearization of the nonlinear oil-film forces in the surroundings of the instant revolutions [4] the obtained equations of motion around the static equilibrium position will be the following:

\[
\begin{align*}
\dot{m}y_1 &= F_y(y_1, \dot{y}_b, \dot{z}_b, y_b, z_b, y, z) + me_1\omega^2\cos(\omega t), \\
\dot{m}z_1 &= F_z(y_1, \dot{y}_b, \dot{z}_b, y_b, z_b, y, z) + me_2\omega^2\sin(\omega t),
\end{align*}
\]

(1)

where \(y_1, z_1, \dot{y}_1, \dot{z}_1\) are displacements, speeds and accelerations of shaft journal centre in the horizontal and vertical vibration plane, respectively (Figure 2), \(y_b, z_b, \dot{y}_b, \dot{z}_b\) are displacements and speeds of bearing bushes centre in the horizontal and vertical vibration plane, \(F_y\) is the horizontal and \(F_z\) vertical component of the oil film force, \(m\) is the shaft mass, \(e_1\) is the shaft imbalance, \(\omega\) is the angular velocity of the shaft rotation, \(t\) is time and \(g\) is the gravitational acceleration. In the case of an uncontrolled rotor system \(y_1, z_1, \dot{y}_1, \dot{z}_1\) are equal to zero.

After the linearization of the nonlinear oil-film forces in the surroundings of the instant revolutions [4] the obtained equations of motion around the static equilibrium position will be the following:

\[
\begin{align*}
\dot{m}\ddot{y}_1 + B_{yy}\dddot{y}_1 + B_{yz}\dddot{z}_1 + K_{yy}\ddot{y}_1 + K_{yz}\ddot{z}_1 + B_{yy}\ddot{y}_b + B_{yz}\ddot{z}_b &= B_{yy}\dddot{y}_b + B_{yz}\dddot{z}_b, \\
\dot{m}\ddot{z}_1 + B_{zy}\dddot{y}_1 + B_{zz}\dddot{z}_1 + K_{zy}\ddot{y}_1 + K_{zz}\ddot{z}_1 + B_{zy}\ddot{y}_b + B_{zz}\ddot{z}_b &= B_{zy}\dddot{y}_b + B_{zz}\dddot{z}_b + me_1\omega^2\cos(\omega t) \\
\end{align*}
\]

(2)

where

\[
\begin{align*}
B_{yy} &= \frac{\partial F_y}{\partial y}, B_{yz} = \frac{\partial F_y}{\partial z}, \\
B_{zy} &= \frac{\partial F_z}{\partial y}, B_{zz} = \frac{\partial F_z}{\partial z}, \\
K_{yy} &= \frac{\partial F_y}{\partial \dot{y}}, K_{yz} = \frac{\partial F_y}{\partial \dot{z}}, \\
K_{zy} &= \frac{\partial F_z}{\partial \dot{y}}, K_{zz} = \frac{\partial F_z}{\partial \dot{z}}.
\end{align*}
\]

(3)

are the damping and stiffness coefficients.

The linearized motion equations can be used for the determination of the stability limit of the rotor system, static equilibrium position and they are suitable for the solution of small vibration in case of rotor revolutions under the limit of the whirl-type instability.

To determine the stability limit the Hurwitz stability criterion can be used. From the linearized motion equations (2) with zero right hand side, the characteristic multinomial of the \(s\) argument can be obtained in the form:

\[
N_p(s) = a_4s^4 + a_3s^3 + a_2s^2 + a_1s + a
\]

\[
a_4 = m^2, a_1 = m(B_{yy} + B_{zz}), \\
a_3 = B_{yy}B_{zz} - B_{yz}B_{zy} + m(K_{yy} + K_{zz}), \\
a_2 = K_{yy}B_{zz} - K_{yz}B_{zy} + K_{zy}B_{yz} + K_{zz}B_{yy}, \\
a_1 = K_{yy}K_{zz} - K_{yz}K_{zy}.
\]

(5)

and the Hurwitz determinant

\[
H = \begin{bmatrix}
a_1 & a_2 & a_3 & a_4 \\
0 & a_1 & a_2 & a_3 \\
0 & 0 & a_1 & a_2 \\
0 & 0 & 0 & a_1
\end{bmatrix}
\]

(6)

**NUMERICAL SIMULATIONS**

Data corresponding to the test stand of the company TECHLAB, Ltd. were used [5]. The journal bearings were approximated as short bearings with following parameters: bearing length 15 mm, bearing radius 15,01 mm, shaft journal radius 14.97 mm, dynamic oil viscosity 0.004 Pa·s and ambient pressure 0 Pa·s.

According to the Hurwitz stability criterion (6) the revolutions at the stability limit 12980 rpm were determined.

After the calculation of bearing forces and after their linearization the damping and stiffness coefficients for revolutions under stability limit were obtained.

To examine the possibilities of the active reduction of rotor lateral vibration the uncontrolled rotor system model and models of rotor systems with feedback P, and PD controllers (Figure 3) were created in the software MATLAB-Simulink. The input data for the revolutions 2000 rpm, 4000 rpm and 6000 rpm and for unbalance mass of 0,001 kg·m can be found in Table 1.

Two time dependences of obtained vibration results in vertical direction at 4000 rpm for the rotor system, one without control and the other with a P controller (\(k_p = 50\)) are compared in Figure 4. (mind the big difference

**Table 1. Parameters of the investigated rotor system**

<table>
<thead>
<tr>
<th>Parameters</th>
<th>2000</th>
<th>4000</th>
<th>6000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Revolutions</td>
<td>/rpm</td>
<td>/rpm</td>
<td>/rpm</td>
</tr>
<tr>
<td>(B_{yy}/kg/s)</td>
<td>4,984×10²</td>
<td>4,844×10²</td>
<td>4,815×10²</td>
</tr>
<tr>
<td>(B_{yz}/kg/s)</td>
<td>-1,129×10³</td>
<td>-5,716×10²</td>
<td>-3,820×10²</td>
</tr>
<tr>
<td>(B_{zy}/kg/s)</td>
<td>-1,129×10³</td>
<td>-5,716×10²</td>
<td>-3,820×10²</td>
</tr>
<tr>
<td>(B_{zz}/kg/s)</td>
<td>5,535×10³</td>
<td>4,989×10³</td>
<td>4,880×10³</td>
</tr>
<tr>
<td>(K_{yy}/kg/s)</td>
<td>2,361×10⁶</td>
<td>2,393×10²</td>
<td>2,400×10²</td>
</tr>
<tr>
<td>(K_{yz}/kg/s)</td>
<td>4,993×10⁶</td>
<td>1,002×10⁷</td>
<td>1,505×10⁷</td>
</tr>
<tr>
<td>(K_{zy}/kg/s)</td>
<td>-6,023×10⁶</td>
<td>-1,057×10⁷</td>
<td>-1,541×10⁷</td>
</tr>
<tr>
<td>(K_{zz}/kg/s)</td>
<td>1,365×10⁵</td>
<td>1,247×10⁵</td>
<td>1,223×10⁵</td>
</tr>
<tr>
<td>(me\omega^2/N)</td>
<td>4,40</td>
<td>17,50</td>
<td>39,40</td>
</tr>
<tr>
<td>(m/kg)</td>
<td>0,3875</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

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The vibration results in horizontal direction, at 4000 rpm for the rotor system without control and with PD controllers ($k_p = 50$, $k_d = 5$ s) are compared in Figure 5. The orbits for a rotor system without control and the one with P controller are compared, (in different scales), in Figure 6.

The summary of the obtained results is in Table 2.

**CONCLUSIONS**

The performed analytical study and numerical simulations have shown, that by active kinematic excitation of the bushes of the bearings, it is possible to reduce significantly the rotor lateral vibration, in the case of a rotor

<table>
<thead>
<tr>
<th>Controller</th>
<th>Revolutions /rpm</th>
<th>Direction /-</th>
<th>Magnitude /m</th>
<th>Equilibrium position /m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Without controller</td>
<td>2000</td>
<td>$y$</td>
<td>$8.2 \times 10^{-6}$</td>
<td>$6.9 \times 10^{-6}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$z$</td>
<td>$8.2 \times 10^{-6}$</td>
<td>$-1.6 \times 10^{-6}$</td>
</tr>
<tr>
<td></td>
<td>4000</td>
<td>$y$</td>
<td>$1.7 \times 10^{-5}$</td>
<td>$3.7 \times 10^{-5}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$z$</td>
<td>$1.7 \times 10^{-5}$</td>
<td>$-4.4 \times 10^{-7}$</td>
</tr>
<tr>
<td></td>
<td>6000</td>
<td>$y$</td>
<td>$2.6 \times 10^{-5}$</td>
<td>$2.5 \times 10^{-5}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$z$</td>
<td>$2.6 \times 10^{-5}$</td>
<td>$-2.0 \times 10^{-7}$</td>
</tr>
<tr>
<td>P controller</td>
<td>2000</td>
<td>$y$</td>
<td>$1.6 \times 10^{-7}$</td>
<td>$2.4 \times 10^{-7}$</td>
</tr>
<tr>
<td>$k_p = 50$</td>
<td></td>
<td>$z$</td>
<td>$1.6 \times 10^{-7}$</td>
<td>$-8.4 \times 10^{-7}$</td>
</tr>
<tr>
<td></td>
<td>4000</td>
<td>$y$</td>
<td>$3.4 \times 10^{-7}$</td>
<td>$2.3 \times 10^{-7}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$z$</td>
<td>$3.4 \times 10^{-7}$</td>
<td>$-7.4 \times 10^{-7}$</td>
</tr>
<tr>
<td></td>
<td>6000</td>
<td>$y$</td>
<td>$5.1 \times 10^{-7}$</td>
<td>$2.6 \times 10^{-7}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$z$</td>
<td>$5.1 \times 10^{-7}$</td>
<td>$-7.0 \times 10^{-7}$</td>
</tr>
<tr>
<td>PD controller</td>
<td>2000</td>
<td>$y$</td>
<td>$2.8 \times 10^{-9}$</td>
<td>$2.4 \times 10^{-9}$</td>
</tr>
<tr>
<td>$k_p = 50$</td>
<td></td>
<td>$z$</td>
<td>$2.8 \times 10^{-9}$</td>
<td>$-8.4 \times 10^{-9}$</td>
</tr>
<tr>
<td>$k_d = 5$ s</td>
<td>4000</td>
<td>$y$</td>
<td>$5.6 \times 10^{-9}$</td>
<td>$2.3 \times 10^{-9}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$z$</td>
<td>$5.6 \times 10^{-9}$</td>
<td>$-7.4 \times 10^{-9}$</td>
</tr>
<tr>
<td></td>
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<td>$y$</td>
<td>$8.5 \times 10^{-9}$</td>
<td>$2.6 \times 10^{-7}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$z$</td>
<td>$8.5 \times 10^{-9}$</td>
<td>$-7.0 \times 10^{-7}$</td>
</tr>
</tbody>
</table>
supported by journal bearings excited by the unbalance in the range of working revolutions, i.e. under the instability limit.

In the case of a two-dimensional control circuit with two identical P controllers there was a reduction of steady-state vibration amplitude in both directions. For 2000 rpm, 4000 rpm and 6000 rpm, and for the chosen controller constant of $k_p = 50$ the vibration amplitude was smaller approximately 51 times. In the case of two identical PD controllers the reduction rate was approximately 300, for the steady-state forced vibration amplitude at 2000 rpm, 4000 rpm and 6000 rpm, with the used controller constants $k_p = 50$ and $k_d = 5s$.

REFERENCES


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