ROBUST CONTROL WITH ENLARGED INTERVAL OF UNCERTAIN PARAMETERS

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Summary Robust control is advantageous for systems with defined interval of uncertain parameters. This can be substantially enlarged dividing it into a few sub-intervals. Corresponding controllers for each of them may be set after approximate identification of some uncertain plant parameters. The paper deals with application of the pole region assignment method for position control of the crane crab. The same track form is required for uncertain burden mass and approximate value of rope length. Measurement of crab position and speed is supposed, burden deviation angle is observed. Simulation results have verified feasibility of this design procedure.

Keywords: robust control, pole region assignment method, observer, crane crab drive.

1. INTRODUCTION

Classical methods of control design assume full knowledge of the system structure and its constant parameters. If this condition is not fulfilled, robust control is recommendable for design with uncertain parameters varying in the given interval [1]. However, this method is not able to ensure the required form of transients within a large interval. This paper presents one possible solution based on creation of a few sub-intervals with robust controllers. The corresponding controller coefficients are adapted by means of the one step approximate identification of the uncertain parameter.

In the considered application the crane crab has to fulfill following requirements: the reference position should be reached with satisfactory exactness, an induction motor fed from a frequency converter is assumed and swinging in the final position is not allowed. Producer defines burden mass of the crane crab within the large interval <0.3, 20-ton> and rope length is supposed to be approximately 4 meters.

2. ROBUST CONTROL DESIGN

System consisting of the converter, induction motor and mechanical part may be described by the state equation

\[ \dot{x} = A(x,p) + B(x,p)u, \]

\[ y = c^T x \]

It contains vector of uncertain parameters

\[ p = [p_1, p_2, ..., p_m]^T. \]

The control function

\[ u = -r^T x \]

with constant feedback parameters \( r_i \) have to be found to ensure placement of moving poles of the characteristic polynomial within the defined \( \Gamma \)-region characterised by stability and the chosen damping. Condition of \( \Gamma \)-stability according to the pole region assignment method [2] valid for the characteristic polynomial \( P(s) = s^2 + a_1 s + a_0 \) with a complex pair of poles \( \lambda_{1,2} = \sigma(\alpha) \pm j \omega(\alpha) \) is given by solution of the following equations:

\[ \begin{bmatrix} d_o(\alpha) & d_1(\alpha) & \cdots & d_n(\alpha) \\ 0 & d_o(\alpha) & \cdots & d_{n-1}(\alpha) \end{bmatrix} \xi(\alpha) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \]

(3)

where \( d_o(\alpha) = 1 \), \( d_i(\alpha) = 2\sigma(\alpha) \), \( d_{i+} = 2\sigma(\alpha) d_i(\alpha) - 2\sigma^2(\alpha) + \omega^2(\alpha) \), for \( i = 1,2,...,n-1 \).

If n-2 feedback parameters are given, a graphical determination of remaining two parameters \( r_1, r_2 \) is possible. The hyperbole equation defining \( \Gamma \)-region is substituted for \( \omega \) into equation (3) and \( \sigma \) is expressed by the generalised frequency \( \alpha = \alpha_1, \alpha_2 \). Then curves for minimal and maximal varying parameters are calculated and their common area determines a possible operating point (Fig.1).
3. Robust control of the crane crab

Robust control design of an induction motor has been presented in [3], similarly as its application for the drive of a crane crab within a limited interval of system parameters [5]. Therefore only concise description of corresponding equations is introduced below.

The mechanical part of a crane crab is shown in Fig. 2, which also contains notation of particular variables. Crab and burden variables are denoted by indices $K$ and $G$ respectively. Motor drives the crab mass $m_K$ by force $F_K$. Force $F_G$ in rope is excluded. Instead of deviation angle $\alpha$ the corresponding arc $\beta$ is considered. The mechanical system is described by means of following differential equations [1], [3]:

$$m_K \ddot{x}_K - m_G g \frac{\beta}{d} = F_K.$$  \hspace{1cm} (4)

$$m_K \ddot{\beta} + (m_K + m_G) g \frac{\beta}{d} = -F_K.$$  \hspace{1cm} (5)

Assuming small deviation angle $\alpha$ the above equations may be linearized. Load torque $m_z$ is expressed by means of the gear ratio $j$, drive wheel radius $r$ and force $F_K$, so that dynamical equation of an induction motor is expressed as [6]:

$$\frac{J}{p} \frac{d\omega}{dt} = \frac{3p}{2} L_h \frac{1}{1 + \sigma_2} i_{2m} i_{1y} - \frac{r}{j} F_K.$$ \hspace{1cm} (6)

$i_{2m}$, $i_{1y}$, $L_h$, $p$, $\sigma_2$ are flux and torque creating current components, main inductance, pair of poles and coefficient of rotor leakage, respectively. After substitution of (6) into (4) and (5) the resulting equations for crab acceleration $\ddot{x}_K$ and angular acceleration $\ddot{\beta}$ are as follows:

$$\ddot{x}_K = a_2 i_{2m} i_{1y} + \frac{a_3}{z} \beta,$$  \hspace{1cm} (7)

$$\ddot{\beta} = -a_3 \frac{a_2}{z} i_{2m} i_{1y} + c \frac{a_3}{z} \beta,$$  \hspace{1cm} (8)

where $c = \frac{J}{p r^2 m_K}$, $e_3 = \frac{J}{p r^2 m_K}$, $z = \frac{d}{g}$

$$a = \frac{m_G}{m_K}, \ a_1 = 1 + a, \ a_3 = \frac{3p}{2} \frac{L_h}{1 + \sigma_2}.$$  \hspace{1cm} (9)

Two feedback gains in Fig.1 were calculated for the first sub-interval with mass $m_G < 0.3$, $5t > t$. Fig.3 depicts movement of system poles within the $\Gamma$-region during parameter variations. Controller parameters of two further sub-intervals $m_G < 5$, $12 > t$ and $m_G < 12$, $20 > t$ were
calculated by the same procedure. Control structure consists of the super-imposed position controller and inner speed and current controllers of the induction motor (Fig. 4).

One-step identification is based on the measured values of motor current and crab speed. At the beginning of crab movement the acceleration $\dot{x}$ has already reached a certain value but speed and arc $\beta$ are still very small. Therefore the second term in (4) may be omitted and dynamical equation (6) in this case is

$$\frac{J_c}{p} \frac{da}{dt} = \frac{3p}{2} \frac{L_h}{1+\sigma_2} \frac{I_y}{I_{2m l y}} - \frac{r}{J} m_K \dot{x}_K. \tag{9}$$

Because the total moment of inertia equals

$$J_c = J_m + (m_K + m_G) \left( \frac{v_K}{\omega_K} \right)^2 \frac{1}{\eta}, \tag{10}$$

the approximate value of the burden mass $m_0$ may be calculated. Its knowledge enables to determine the corresponding sub-interval and set of controller parameters.

Measurement of the arc $\beta$ corresponding to the burden deviation from the perpendicular position is difficult because more complex sensor would be necessary. Therefore a linear observer with adapted parameters was designed [4]:

$$\ddot{x} = Ax(p) + bu + h(y - \ddot{y}), \tag{11}$$

$$h = \left[ f \ I + f \ A + \ldots + f \ A^{-} + A^{-} \right] q, \tag{12}$$

where $f_i$ are coefficients of the desired characteristic polynomial and $q$ is the last column of the inverse observability matrix.

4. SIMULATION RESULTS

Robust controllers of the crane crab were calculated by MAPLE and verified by simulation using MATLAB programme.

![Graph showing time responses of crab position at 20 000 kg](image_url)
Fig. 6. Time responses of crab position at 12 000 kg.

Fig. 7. Desired rope length at various burden masses

Insensitivity to the rope length at uncertain burden in the first sub-interval may be ensured within the area B depicted in Fig.7. If value of the rope length would be chosen outside of these limits (Fig.8), damping could be minor but crab movement still remains stable.

5. CONCLUSION

Robust control is very advantageous for less pretentious drives with varying or approximately determined system parameters calculated from the nameplate or catalogue data. However, the broader interval of parameters is taken into consideration the slower dynamics have to be expected, because constant feedback gains must correspond to the worst case. This disadvantage may be reduced by choice of a few narrower sub-intervals with controller parameters adapted after approximate and simple identification. An observer with adapted parameter estimates the difficult measurable system parameter.

REFERENCES