DIAGNOSTICS AND RECONFIGURATION OF CONTROL SYSTEMS

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Summary This contribution summarizes some of the major trends, as well as real opportunities for the application of system fault diagnosis and reconfiguration control structures in automatic control systems. There are referenced ongoing concepts for residual generation, described design approaches in virtual reconfiguration and, in more details, is presented a new method for robust structured residual design and one another for reconfigurable output control structure.

1. INTRODUCTION

The complexity of control systems requires fault tolerance schemes to provide control of the faulty system. Fault tolerant systems are that one of the more fruitful applications with potential significance for domains in which control of systems must proceed while the system is operative and testing opportunities are limited by operational considerations. The real problem is usually to fix the system with faults so that it can continue its mission for some time with some limitations of functionality.

Automated diagnosis is one part of these large problems known as fault detection, identification and reconfiguration (FDIR). The practical benefits of an integrated approach to FDIR seem to be considerable, especially when knowledge of available fault isolations and system reconfiguration is used to reduce the cost and increase the reliability and utility of control.

The paper presents some directions in the field of dynamic system fault diagnosis and control structure reconfiguration design, with special emphasis on structured residual generator design for systems with unknown input disturbance, as well as on the reconfiguration flexibility offered by state-space feedback control.

2. DIAGNOSIS AND RECONFIGURATION

The essential aspect for the design of fault-tolerant control requires the conception of diagnosis procedure that can solve the fault detection and isolation problem. This procedure composes residual signal generation (signals that contain information about the failures or defects) followed by their evaluation within decision functions.

In principle, in order to achieve fault tolerance, some redundancy is necessary. So far direct redundancy is realized by redundancy in multiple hardware channels, fault-tolerant control involve functional redundancy. Functional (analytical) redundancy is usually achieved by design of such subsystems, which functionality is derived from system model and can be realized using algorithmic (software) redundancy. Thus, analytical redundancy most often means the use of functional relations between system variables and residuals are derived from implicit information in functional or analytical relationships, which exist between measurements taken from the process, and a process model. In this sense a residual is a fault indicator, based on a deviation between measurements and model-equation-based computation and model based diagnosis uses models to obtain residual signals that are as a rule zero in the fault free case and non-zero otherwise.

The main goal when synthesizing robust residual generators for diagnosis and supervision, as well as robust control algorithms, is to attenuate influence from model uncertainty while keeping fault detection and control performance. Since available models of real processes always are uncertain, there is naturally a need for robust methods minimizing the sensitivity to the model uncertainties and disturbances.

A fault in the fault diagnosis systems can be detected and located when has to cause a residual change and subsequent analyze of residuals have to provide information about faulty component localization. The diagnosis is so a decision process (pattern recognition process) whose goal is to decide whether fault is present or not (to classify pattern by computing them to prototypes given by the set of classes). From this point of view the fault decision information is capable in a suitable format to specify possible control structure class to facilitate the appropriate adaptation of the control feedback laws.

The main task to be tackled in achieving fault-tolerance is the design a controller with suitable reconfigurable structure to guarantee stability, satisfactory performance and plant operation economy in nominal operational conditions, but also in some components malfunction. Thus, fault-tolerant control is a strategy for reliable and highly efficient control law design and includes fault-tolerant system requirements analysis, analytical redundancy design (fault isolation principles) and fault accommodation design (fault control requirements and reconfigurable control strategy).

Whereas diagnosis is the problem of identifying elements whose abnormality is sufficient to explain an observed malfunction, reconfiguration can be viewed as the problem of identifying elements whose reconfiguration is sufficient to restore acceptable behavior of the system. Used reconfigurable strategies are derived from systems, which do not posses considerable hardware redundancy and system properties can be changed by control algorithm (structure) modification. The approach follows from the insight, that reconfiguration can be viewed as the problem of specifying structures (in limited case the controller elements) whose reconfiguration is sufficient to restore acceptable behavior for acceptable faults. The benefits result from this characterization give a unified framework that should facilitate the development of an integrated theory of FDIR and control (fault-tolerant control).

Passive approaches to fault-tolerant control make use of robust control technique to ensure that a closed-loop system remains insensitive to certain faults using constant controller parameters and without use of on-line fault information. Active fault-tolerant control requires a mechanism for detecting and isolating unanticipated abnormal system changes to reschedule controller function. Thus, system reconfigurability implies that fixed structure can be modified to account for uncontrollable changes in the system, i.e. active fault-tolerant controllers are generally variable in their structure, but use the concept of unanticipated faults. Modern methods for reconfigurable control design have to take into account nominal system parameters, and include faults residual effects as well as modeling errors and inaccuracies of the fault decision and system models in FDIR system robustness.

3. THE STATE OF THE ART

Model-based fault diagnosis can be understood as the detection, isolation and determination of faults in components of a system from comparison of its available measurements with a priori information represented by the system's mathematical model. Faults are detected usually by setting a threshold on a residual signal generated from the difference between real measurements and their estimates using the mathematical model. The major sub-classes of model-based FDI, based on quantitative models, are parity equation, state estimation, and parameter estimation approaches, respectively.

The systems under consideration can be understood as multi-input and multi-output linear (MIMO) dynamic system with unknown input disturbance and in state-space form this class of discrete-time linear dynamic system be represented as

$$\boldsymbol{q}(i+1) = \boldsymbol{F}\boldsymbol{q}(i) + \boldsymbol{G}(\boldsymbol{u}(i) + \boldsymbol{f}_a(i)) + \boldsymbol{E}\boldsymbol{d}(i) \quad (1)$$

$$\boldsymbol{y}(i) = \boldsymbol{C}\boldsymbol{q}(i) + \boldsymbol{f}_s(i) \quad (2)$$

where $\boldsymbol{q}(i) \in \mathbb{R}^n$, $\boldsymbol{u}(i) \in \mathbb{R}^r$ and $\boldsymbol{y}(i) \in \mathbb{R}^m$ are vectors of the state, input and output variables, respectively, and $\boldsymbol{F} \in \mathbb{R}^{n\times n}$, $\boldsymbol{G} \in \mathbb{R}^{n\times r}$, $\boldsymbol{C} \in \mathbb{R}^{m\times n}$ are real matrices of full ranks. Monitored faults in the system actuators and system sensors are modeled by two additive vectors $\boldsymbol{f}_a(i) \in \mathbb{R}^r$, $\boldsymbol{f}_s(i) \in \mathbb{R}^m$. In the next, it is supposed that system input uncertainties are structured, i.e. it is known how they enter the system dynamics through appropriated matrices $\boldsymbol{E}$ and, in general, the unknown disturbances $\boldsymbol{d}(i)$ acting on the system can includes the non-monitored system faults.

The basic idea of the parity relations approach is to provide a proper check of the parity (consistency) of the measurement acquired from the monitored system. For a system without disturbance the generalized parity space equation is

$$\boldsymbol{y}_Y(i) = \boldsymbol{Q}_P\boldsymbol{q}(i-h) + \boldsymbol{Q}_U(\boldsymbol{u}_U(i) + \boldsymbol{f}_{Fa}(i)) + \boldsymbol{f}_{Fs}(i) \quad (3)$$

where $\boldsymbol{Q_P}$ is the observability matrix and $\boldsymbol{Q_U}$ is the Toeplitz matrix of the Markov system parameters of appropriate dimensions defined by *m*, *n*, and *h*, and realized residual vector equation can be chosen as

$$\boldsymbol{r}(i) = \boldsymbol{V}_M(\boldsymbol{y}_Y(i) - \boldsymbol{Q}_U\boldsymbol{u}_U(i)) \quad (4)$$

To obtain residual vector decoupled from state variable vector $\boldsymbol{q}(i\text{-}h)$, the projection matrix $\boldsymbol{V}_M$ need to satisfy condition

$$\boldsymbol{V}_M\boldsymbol{Q}_P = \boldsymbol{0} \quad (5)$$

This leads to vector residual

$$\boldsymbol{r}(i) = \boldsymbol{V}_M\boldsymbol{Q}_U\boldsymbol{f}_{Fa}(i) + \boldsymbol{V}_M\boldsymbol{f}_{Fs}(i) \quad (6)$$

One method to solve (5) is presented e.g. in [8].

The basic idea behind the observer and filter-based technique is to estimate the outputs of the system from the measurement by using either Luenberger estimator or Kalman predictor.

Assuming that matrices $\boldsymbol{F}$, $\boldsymbol{G}$, and $\boldsymbol{C}$ are known, and $\boldsymbol{E} = \boldsymbol{0}$, the estimator equations for a system (1), (2) are

$$\boldsymbol{q}_e(i+1) = \boldsymbol{F}\boldsymbol{q}_e(i) + \boldsymbol{G}\boldsymbol{u}(i) + \boldsymbol{J}(\boldsymbol{y}(i) - \boldsymbol{y}_e(i)) \quad (7)$$

$$\boldsymbol{y}_e(i) = \boldsymbol{C}\boldsymbol{q}_e(i) \quad (8)$$

Design task is to determine matrix $\boldsymbol{J}$ by that way, that all eigenvalues of the matrix $\boldsymbol{F}_e = \boldsymbol{F} - \boldsymbol{JC}$ are from the unit circle centered at the origin of the complex plane z. Since for MIMO systems and for prescribed set of desired eigenvalues this solution is not unique, a design method based on singular value decomposition (SVD) can be found e.g. in [6].

Then, the output estimation error can be used as residual, i.e.

$$\boldsymbol{r}(i) = \boldsymbol{y}(i) - \boldsymbol{y}_e(i) \quad (9)$$

There exist robust and structured modifications of this principle, which can be found e.g. in [3], [8].

Kalman predictor equations, and derived residuals, are given in the same structure as (7) to (9). The gain matrix $\boldsymbol{J}$ is computed as

$$\boldsymbol{J} = (\boldsymbol{FPC}^T + \boldsymbol{S})(\boldsymbol{R} + \boldsymbol{CPC}^T)^{-1} \tag{10}$$

where $\boldsymbol{P}$ is a solution of the algebraic discrete Riccati equation

$$\begin{aligned} \boldsymbol{P} &= \boldsymbol{FPF}^T + \boldsymbol{Q} - \\ &-(\boldsymbol{FPC}^T + \boldsymbol{S})(\boldsymbol{R} + \boldsymbol{CPC}^T)^{-1}(\boldsymbol{FPC}^T + \boldsymbol{S})^T \end{aligned} \tag{11}$$

and $\boldsymbol{P}$, $\boldsymbol{Q}$, and $\boldsymbol{R}$ are real symmetric positive definite matrices of appropriated dimension. Using in noise environment, an algorithm to estimate the system and the measurement noise covariance $\boldsymbol{Q}$, $\boldsymbol{R}$, and $\boldsymbol{S}$ is presented e.g. in [5]. Since a threshold setting on the residual signal in a noise environment can call false alarms, evaluation is based on the residual mean-value change detection using the Shewhart graph algorithm modification (see e.g. [1], [8]).

In most practical cases, the process parameters are not known at all and they can be determined with parameter estimation methods. The least-square estimation of the SISO system parameters (with exponential forgetting) can be expressed in recursive (Kalman) form [8], [13]

$$\boldsymbol{d}(i+1) = \boldsymbol{d}(i) + \boldsymbol{j}(i+1)(y(i+1) - y_e(i+1)) \tag{12}$$

$$y_e(i+1) = \boldsymbol{l}^T(i+1)\boldsymbol{d}(i) \tag{13}$$

$$\boldsymbol{j}(i+1) = \boldsymbol{P}(i)\boldsymbol{l}(i+1)r^{-1}(i+1) \tag{14}$$

$$r(i+1) = b^2 + \boldsymbol{l}^T(i+1)\boldsymbol{P}(i)\boldsymbol{l}(i+1) \tag{15}$$

$$\boldsymbol{P}(i+1) = \boldsymbol{P}(i)r^{-1}(i+1) \tag{16}$$

$$\begin{aligned} &\boldsymbol{l}^T(i) = \\ &= \left[-y(i-1)\cdots -y(i-n)\, u(i-1)\cdots u(i-n)\right] \end{aligned} \tag{17}$$

$$\boldsymbol{d}(i) = \left[\, f_{n-1}\cdots f_0\; g_{n-1}\cdots g_0 \right]^T \tag{18}$$

where $b \in (0, 1\rangle$ is a forgetting factor, $\boldsymbol{P}$ is a symmetric positive definite matrix and $\boldsymbol{d}$ is the system parameter vector. Then, an estimation error is used as residual, i.e.

$$\boldsymbol{r}(i) = \boldsymbol{d} - \boldsymbol{d}(i) \tag{19}$$

and its evaluation is realized using the mean-value change detection. For improved estimate of MIMO system parameters, subspace identification methods are used [10], [12].

Many modifications of above presented methods are known especially for residual filter design using linear/nonlinear continuous-time system models (see e.g. [2], [4], [8]).

To achieve fault tolerance used methods relies on employing on-line fault diagnosis schemes, react to the results of diagnosis and activate an alternative control (reconfigurable control structure) that is supposed to handle the fault. Among these structures can be quoted control systems with adaptation to faults, the virtual-based control structures, as well as output control reconfiguration algorithms, which guarantee the dominant closed-loop dynamics.

Adaptation to faults is one of the earliest methods for the controller re-design, generally based on the model-matching. Hard limitation implies from condition that all closed loop have to take the similar dynamics. One new way to construct a reconfigurable control with adaptation to sensor faults is presented in [9].

Opposite strategies use virtual based reconfigurable principle. Instead of adapting the controller to the faulty system a reconfiguration goal is virtually adapt faulty system to the nominal controller. The virtual sensor is generally based on the Luenberger estimator and the virtual actuator takes its dual form.

In the case of sensor faults virtual sensor can be designed in the form

$$\begin{aligned} &\boldsymbol{q}_e(i+1) = \\ &= \boldsymbol{Fq}_e(i) + \boldsymbol{Gu}_f(i) + \boldsymbol{J}(\boldsymbol{y}_f(i) - \boldsymbol{C}_f\boldsymbol{q}_e(i)) \end{aligned} \tag{20}$$

$$\boldsymbol{y}_e(i) = \boldsymbol{Xy}_f(i) + (\boldsymbol{C} - \boldsymbol{XC}_f)\boldsymbol{q}_e(i) \tag{21}$$

where $\boldsymbol{C}_f$ is the output matrix of the system with a sensor fault and $\boldsymbol{y}_f$ (i) is the faulty measurement vector at time instant i. If $\boldsymbol{X} = \boldsymbol{0}$, estimated vector is used for control, if $\boldsymbol{X} = \boldsymbol{I}$, the outputs of fault-free sensors are combined with associated estimate, to substitute missing output of the faulty sensor. The duality in virtual actuator design one can see in [2].

4. SOME NEW SOLUTIOS

4.1 Robust Structured Residuals

Under assumption that matrices $\boldsymbol{F}$, $\boldsymbol{E}$, and $\boldsymbol{C}$ are known, a set of structured estimators with respect to system outputs can be designed to a system (1), (2), where $k = 1,2, \ldots, m$, and

$$\boldsymbol{p}_k(i+1) = \boldsymbol{P}_k\boldsymbol{p}_k(i) + \boldsymbol{Q}_k\boldsymbol{Gu}(i) + (\boldsymbol{J}_k + \boldsymbol{K}_k)\boldsymbol{T}_k\boldsymbol{y}(i) \tag{22}$$

$$\boldsymbol{q}_{ek}(i) = \boldsymbol{p}_k(i) + \boldsymbol{O}_k\boldsymbol{T}_k\boldsymbol{y}(i) \tag{23}$$

Here $\boldsymbol{p}_k(i) \in \mathbb{R}^n$ is the state vector of the k-th estimator, $\boldsymbol{q}_{ek}(i) \in \mathbb{R}^n$ is a system state vector $\boldsymbol{q}(i)$ estimate derived from the k-th estimator state vector, $\boldsymbol{P}_k \in \mathbb{R}^{n\times n}$, $\boldsymbol{Q}_k \in \mathbb{R}^{m\times n}$, $\boldsymbol{J}_k \in \mathbb{R}^{n\times(m-1)}$, $\boldsymbol{K}_k \in \mathbb{R}^{n\times(m-1)}$, as well as $\boldsymbol{O}_k \in \mathbb{R}^{n\times(m-1)}$ are designed matrix parameters and matrix $\boldsymbol{T}_k = \boldsymbol{I}_{m\varnothing k} \in \mathbb{R}^{(m-1)\times m}$ is degenerative identity matrix, which k-th row is deleted.

Then, with absence of faults, the state estimation error can be expressed as follows

$$\begin{aligned} &\boldsymbol{e}_k(i+1) = \boldsymbol{q}(i+1) - \boldsymbol{q}_{ek}(i+1) = \\ &= \boldsymbol{P}_k\boldsymbol{e}_k(i) + (\boldsymbol{F} - \boldsymbol{P}_k - \boldsymbol{J}_k\boldsymbol{T}_k\boldsymbol{C} - \boldsymbol{O}_k\boldsymbol{T}_k\boldsymbol{CF})\boldsymbol{q}(i) + \\ &+(\boldsymbol{I}_n - \boldsymbol{Q}_k - \boldsymbol{O}_k\boldsymbol{T}_k\boldsymbol{C})\boldsymbol{Gu}(i) + (\boldsymbol{P}_k\boldsymbol{O}_k - \boldsymbol{K}_k)\boldsymbol{T}_k\boldsymbol{y}(i) + \\ &+(\boldsymbol{I}_n - \boldsymbol{O}_k\boldsymbol{T}_k\boldsymbol{C})\boldsymbol{Ed}(i) \end{aligned} \tag{24}$$

where $\boldsymbol{I}_n \in \mathbb{R}^{n\times n}$ is the identity matrix.

It is evident, to obtain an autonomous state estimation error vector, the design conditions have to be

$$\boldsymbol{P}_k = \boldsymbol{F} - \boldsymbol{J}_k\boldsymbol{T}_k\boldsymbol{C}_k - \boldsymbol{O}_k\boldsymbol{T}_k\boldsymbol{CF} \tag{25}$$

$$\boldsymbol{Q}_k = \boldsymbol{I}_n - \boldsymbol{O}_k\boldsymbol{T}_k\boldsymbol{C} \tag{26}$$

$$\boldsymbol{K}_k = \boldsymbol{P}_k\boldsymbol{O}_k \tag{27}$$

$$(\boldsymbol{I}_n - \boldsymbol{O}_k \boldsymbol{T}_k \boldsymbol{C})\boldsymbol{E} = 0 \tag{28}$$

and then the state estimation error difference equation reduces to the form

$$\boldsymbol{e}_k(i+1) = \boldsymbol{P}_k \boldsymbol{e}_k(i) \tag{29}$$

It is evident, to guarantee asymptotic stability, matrix $\boldsymbol{P}_k$ have to be stable.

4.1.1 *Disturbance Decoupling*

The disturbance decoupling can be achieved using condition (28), i.e.

$$\boldsymbol{O}_k \boldsymbol{T}_k \boldsymbol{C}\boldsymbol{E} = \boldsymbol{E} \tag{30}$$

Multiplying (11) on the right side by identity matrix gives

$$\boldsymbol{O}_k = \boldsymbol{E}((\boldsymbol{T}_k\boldsymbol{C}\boldsymbol{E})^T \boldsymbol{T}_k\boldsymbol{C}\boldsymbol{E})^{-1}(\boldsymbol{T}_k\boldsymbol{C}\boldsymbol{E})^T = \boldsymbol{E}(\boldsymbol{T}_k\boldsymbol{C}\boldsymbol{E})^{\Diamond} \tag{31}$$

where

$$(\boldsymbol{T}_k\boldsymbol{C}\boldsymbol{E})^{\Diamond} = ((\boldsymbol{T}_k\boldsymbol{C}\boldsymbol{E})^T \boldsymbol{T}_k\boldsymbol{C}\boldsymbol{E})^{-1}(\boldsymbol{T}_k\boldsymbol{C}\boldsymbol{E})^T \tag{32}$$

is the Penrose pseudoinverse of a matrix $\boldsymbol{T}_k\boldsymbol{C}\boldsymbol{E}$.

Substituting (31) into (30) and multiplying this result on the left side by matrix $\boldsymbol{T}_k\boldsymbol{C}$ one can obtain

$$(\boldsymbol{I}_{m-1} - \boldsymbol{T}_k\boldsymbol{C}\boldsymbol{E}(\boldsymbol{T}_k\boldsymbol{C}\boldsymbol{E})^{\Diamond})\boldsymbol{T}_k\boldsymbol{C}\boldsymbol{E} \doteq 0 \tag{33}$$

and so all solutions of (30) are

$$\boldsymbol{O}_k = \boldsymbol{E}(\boldsymbol{T}_k\boldsymbol{C}\boldsymbol{E})^{\Diamond} + \boldsymbol{O}_{kE}(\boldsymbol{I}_{m-1} - \boldsymbol{T}_k\boldsymbol{C}\boldsymbol{E}(\boldsymbol{T}_k\boldsymbol{C}\boldsymbol{E})^{\Diamond}) \tag{34}$$

where $\boldsymbol{O}_{kE}$ is any nonzero matrix of appropriate dimension.

4.1.2 *Solution of Estimate Error Convergence*

Using (34) the system matrix (25) can be written as

$$\begin{aligned} \boldsymbol{P}_k &= \boldsymbol{F} - \boldsymbol{J}_k\boldsymbol{T}_k\boldsymbol{C}_k - \boldsymbol{O}_k\boldsymbol{T}_k\boldsymbol{C}\boldsymbol{F} = \\ &= (\boldsymbol{I}_n - \boldsymbol{E}(\boldsymbol{T}_k\boldsymbol{C}\boldsymbol{E})^{\Diamond}\boldsymbol{T}_k\boldsymbol{C})\boldsymbol{F} - \boldsymbol{J}_k\boldsymbol{T}_k\boldsymbol{C} - \\ &\quad - \boldsymbol{O}_{kE}\boldsymbol{T}_k\boldsymbol{C}(\boldsymbol{I}_n - \boldsymbol{E}(\boldsymbol{T}_k\boldsymbol{C}\boldsymbol{E})^{\Diamond}\boldsymbol{T}_k\boldsymbol{C})\boldsymbol{F} \end{aligned} \tag{35}$$

$$\boldsymbol{P}_k = \boldsymbol{F}_{k1} - \begin{bmatrix} \boldsymbol{J}_k & \boldsymbol{O}_{kE} \end{bmatrix} \begin{bmatrix} \boldsymbol{T}_k\boldsymbol{C} \\ \boldsymbol{T}_k\boldsymbol{C}\boldsymbol{F}_{k1} \end{bmatrix} = \boldsymbol{F}_{k1} - \boldsymbol{J}_{k1}\boldsymbol{C}_{k1} \tag{36}$$

respectively, where

$$\boldsymbol{F}_{k1} = (\boldsymbol{I}_n - \boldsymbol{E}(\boldsymbol{T}_k\boldsymbol{C}\boldsymbol{E})^{\Diamond}\boldsymbol{T}_k\boldsymbol{C})\boldsymbol{F} \in \mathbb{R}^{n\times n} \tag{37}$$

$$\boldsymbol{J}_{k1} = \begin{bmatrix} \boldsymbol{J}_k & \boldsymbol{O}_{kE} \end{bmatrix} \in \mathbb{R}^{n\times 2(m-1)} \tag{38}$$

$$\boldsymbol{C}_{k1} = \begin{bmatrix} \boldsymbol{T}_k\boldsymbol{C} \\ \boldsymbol{T}_k\boldsymbol{C}\boldsymbol{F}_{k1} \end{bmatrix} \in \mathbb{R}^{2(m-1)\times n} \tag{39}$$

Equation (36) takes the standard structure of the state estimator system matrix and within the design task matrix $\boldsymbol{P}_k$ has to be designed in such a way, that all its eigenvalues be stable. Therefore, the goal is to select a real matrix $\boldsymbol{J}_{k1}$ which can be computed using e.g. the singular-value decomposition (SVD) method for prescribed set of estimator system matrix eigenvalues $\{z_{ki}, |z_{ki}| < 1, i = 1,2, ..., n\}$ [6].

It is obvious, that the first m-1 columns of $\boldsymbol{J}_{k1}$ note the matrix $\boldsymbol{J}_k$, and the rest columns specify the matrix $\boldsymbol{O}_{kE}$. Knowing $\boldsymbol{J}_k$, as well as $\boldsymbol{O}_{kE}$ one can compute $\boldsymbol{O}_k$ from (15), and $\boldsymbol{Q}_k$ and $\boldsymbol{K}_k$ using (7), (8), respectively.

4.1.3 *Structured Residual Design*

Generally, the structured residual vectors $\boldsymbol{r}_k$, for $k = 1,2, \dots, m$, can be defined as

$$\boldsymbol{r}_k(i) = \boldsymbol{X}_k\boldsymbol{q}_{ek}(i) + \boldsymbol{Y}_k\boldsymbol{y}(i) \tag{40}$$

Denoting

$$\boldsymbol{q}_{ek}(i) = \boldsymbol{q}(i) - \boldsymbol{e}_k(i) \tag{41}$$

and injecting (2) and (22) into (21) results in

$$\boldsymbol{r}_k(i) = (\boldsymbol{X}_k + \boldsymbol{Y}_k\boldsymbol{C})\boldsymbol{q}(i) - \boldsymbol{X}_k\boldsymbol{e}_k(i) + \boldsymbol{Y}_k\boldsymbol{f}_s(i) \tag{42}$$

To make residuals decoupled from state vector, it is possible to set

$$\boldsymbol{X}_k = -\boldsymbol{T}_k\boldsymbol{C}, \qquad \boldsymbol{Y}_k = \boldsymbol{T}_k \tag{43}$$

Then a set of structured residual computational equations takes form

$$\boldsymbol{r}_k(i) = \boldsymbol{T}_k\boldsymbol{C}\boldsymbol{e}_k(i) + \boldsymbol{T}_k\boldsymbol{f}_s(i), \quad k = 1,2,\dots,m \tag{44}$$

4.1.4 *Sensor and Actuator Faults Action*

Structured residual generator equation (44) implies, the k-th sensor fault is not observed in the k-th residual since defined basic property of $\boldsymbol{T}_k$. Using (1), (2) and (25) – (28) one can verify that

$$\begin{aligned} &\boldsymbol{e}_k(i+1) = \\ &= \boldsymbol{P}_k\boldsymbol{e}_k(i) + \boldsymbol{Q}_k\boldsymbol{G}\boldsymbol{f}_a(i) - \boldsymbol{J}\boldsymbol{T}_k\boldsymbol{f}_s(i) - \boldsymbol{O}_k\boldsymbol{T}_k\boldsymbol{f}_s(i+1) \end{aligned} \tag{45}$$

and, since (25) implies

$$\boldsymbol{r}_k(i+1) = \boldsymbol{T}_k\boldsymbol{C}\boldsymbol{e}_k(i+1) + \boldsymbol{T}_k\boldsymbol{f}_s(i+1) \tag{46}$$

the computational form of the residual vector can be rewritten as

$$\begin{aligned} &\boldsymbol{r}_k(i+1) = \boldsymbol{T}_k\boldsymbol{f}_s(i+1) + \\ &+ \boldsymbol{T}_k\boldsymbol{C}(\boldsymbol{P}_k\boldsymbol{e}_k(i) + \boldsymbol{Q}_k\boldsymbol{G}\boldsymbol{f}_a(i) - \boldsymbol{J}\boldsymbol{T}_k\boldsymbol{f}_s(i) - \boldsymbol{O}_k\boldsymbol{T}_k\boldsymbol{f}_s(i+1)) \end{aligned} \tag{47}$$

It can be seen in (47) the actuator falts are observed in all residual generators with time-delay equal one period of sampling. Since state error estimate convergence is provided and disturbance is decoupled, the residuals are aproximatly zeros in a fault-free routine.

4.2 Reconfigurable Output Control Structure

Assuming the system is both controllable and observable, as well as the input and output matrices are of full rank, that is $rank(\boldsymbol{G}) = r$, $rank(\boldsymbol{C}) = m$, and $r = m < n$, , $rank(\boldsymbol{F}) = n$, then there exist matrix $\boldsymbol{K}$ such that the static output feedback control law of the form

$$\boldsymbol{u}(i) = -\boldsymbol{K}\boldsymbol{y}(i) = -\boldsymbol{K}\boldsymbol{C}\boldsymbol{q}(i) \tag{48}$$

can be designed.

The freedom that characterizes the placing of the closed-loop system matrix eigenvalues and associated closed-loop eigenvectors by eigenstructure assignment using output feedback means, that

(i) $max(r,m)$ closed-loop eigenvalues can be assigned,

(ii) $max(r,m)$ eigenvectors can be associated with assigned closed-loop eigenvalues.

In view of (1), (2), and (48), the autonomous closed-loop system is described by equatios

$$\boldsymbol{q}(i+1) = (\boldsymbol{F} - \boldsymbol{GKC})\boldsymbol{q}(i) \tag{49}$$

$$\boldsymbol{y}(i) = \boldsymbol{Cq}(i) \tag{50}$$

The solution of the aforementioned problem is a real matrix $\boldsymbol{K} \in \mathbb{R}^{r \times m}$ which can be designed using singular value decomposition (SVD) method for a set of eigenvalues $\{z_i, |z_i| < 1, i = 1,2, ..., m\}$ [6].

4.2.1 *State Vector Transformation*

Using given state model of the system (1), (2), it is possible to transform the state vector $\boldsymbol{q}(i)$ to the input-closed state space by a matrix $\boldsymbol{T}_G \in \mathbb{R}^{n \times n}$ to yield the realization

$$\boldsymbol{q}(i) = \boldsymbol{T}_G \boldsymbol{q}_G(i) \tag{51}$$

Since (51) implies

$$\boldsymbol{T}_G \boldsymbol{q}_G(i+1) = \boldsymbol{F}\boldsymbol{T}_G \boldsymbol{q}_G(i) + \boldsymbol{Gu}(i) \tag{52}$$

this follows as a consequence

$$\boldsymbol{q}_G(i+1) = \boldsymbol{F}_G \boldsymbol{q}_G(i) + \boldsymbol{G}_G \boldsymbol{u}(i) \tag{53}$$

$$\boldsymbol{y}(i) = \boldsymbol{C}_G \boldsymbol{q}_G(i) \tag{54}$$

where

$$\boldsymbol{F}_G = \boldsymbol{T}_G^{-1}\boldsymbol{F}\boldsymbol{T}_G, \quad \boldsymbol{C}_G = \boldsymbol{C}\boldsymbol{T}_G \tag{55}$$

$$\boldsymbol{G}_G = \boldsymbol{T}_G^{-1}\boldsymbol{G} = \begin{bmatrix} \boldsymbol{I}_r \\ \boldsymbol{0}_{n-r,r} \end{bmatrix}, \quad \boldsymbol{T}_G = \left[\boldsymbol{G} \;\middle|\; \begin{matrix} \boldsymbol{I}_r \\ \boldsymbol{0}_{n,n-r} \end{matrix} \right] \tag{56}$$

With (55), (56) control law (48) takes form

$$\boldsymbol{u}(i) = -\boldsymbol{KCT}_G\boldsymbol{T}_G^{-1}\boldsymbol{q}(i) = -\boldsymbol{KC}_G\boldsymbol{q}_G(i) \tag{57}$$

and the closed-loop system equation (49) is transformed to

$$\boldsymbol{q}_G(i+1) = (\boldsymbol{F}_G - \boldsymbol{G}_G\boldsymbol{KC}_G)\boldsymbol{q}_G(i) \tag{58}$$

As one can see, the state transformation does not affect the output feedback gain matrix and this is also true for the eigenvalues of the transformed system.

4.2.2 *Feedback Gain Matrix Design*

For any pair of closed-loop eigenvalues and their associated eigenvectors $\{(z_j, \boldsymbol{n}_j), j = 1,2, ...,m\}$, generally complex conjugate, holds

$$(\boldsymbol{F}_G - \boldsymbol{G}_G\boldsymbol{KC}_G)\boldsymbol{n}_j = z_j\boldsymbol{n}_j \tag{59}$$

where $\boldsymbol{n}_j$ is the j-th right eigenvector. Equality (59) can be rewritten to the singular form

$$\boldsymbol{L}_{Gj}\begin{bmatrix} \boldsymbol{n}_j \\ \boldsymbol{KC}_G\boldsymbol{n}_j \end{bmatrix} = \boldsymbol{0} \tag{60}$$

$$\boldsymbol{L}_{Gj} = \left[z_j\boldsymbol{I} - \boldsymbol{F}_G \;\middle|\; \begin{matrix} \boldsymbol{I}_r \\ \boldsymbol{0}_{n-r,r} \end{matrix} \right] \tag{61}$$

and using SVD procedures applied to all matrices $\boldsymbol{L}_{Gj}$ one can design gain matrix $\boldsymbol{K}$ [9].

4.2.3 *Control Reconfiguration*

The system faults modify the system properties, which can be now described by equations

$$\boldsymbol{q}(i+1) = \boldsymbol{F}_f\boldsymbol{q}(i) + \boldsymbol{G}_f\boldsymbol{u}(i) \tag{62}$$

$$\boldsymbol{y}(i) = \boldsymbol{C}_f\boldsymbol{q}(i) \tag{63}$$

where $\boldsymbol{F}_f$, $\boldsymbol{G}_f$, and $\boldsymbol{C}_f$ are system matrices of the same dimensions with matrices of the nominal model.

The reconfiguration task is to include a new stabilizing feedback control law

$$\boldsymbol{u}(i) = -\boldsymbol{K}_f\boldsymbol{y}(i) = -\boldsymbol{K}_f\boldsymbol{C}_f\boldsymbol{q}(i) \tag{64}$$

in such a way that a new closed-loop system matrix $\boldsymbol{F}_f - \boldsymbol{G}_f\boldsymbol{K}_f\boldsymbol{C}_f$ can capture as much as possible the eigenstructure of the nominal closed-loop system matrix (with the same dominat eigenvalues of matrices). Design is based on the same transformation as (55) and (56) but using $\boldsymbol{T}_{Gf}$ with structure

$$\boldsymbol{G}_{Gf} = \boldsymbol{T}_{Gf}^{-1}\boldsymbol{G}_f = \begin{bmatrix} \boldsymbol{I}_r \\ \boldsymbol{0}_{n-r,r} \end{bmatrix}, \quad \boldsymbol{T}_{Gf} = \left[\boldsymbol{G}_f \;\middle|\; \begin{matrix} \boldsymbol{I}_r \\ \boldsymbol{0}_{n,n-r} \end{matrix} \right] \tag{65}$$

4.2.4 *Optimization*

Transformation matrices $\boldsymbol{T}_G$ or $\boldsymbol{T}_{Gf}$ are not unique and there exist other structures of this matrices given by permutations in rows of the basic structure, i.e.

$$\boldsymbol{T}_G = \left[\boldsymbol{G} \;\middle|\; \begin{matrix} \boldsymbol{I}_r \\ \boldsymbol{0}_{n,n-r} \end{matrix} \right] \approx \boldsymbol{T}_{Gh} = \left[\boldsymbol{G} \;\middle|\; \begin{pmatrix} \boldsymbol{I}_r \\ \boldsymbol{0}_{n,n-r} \end{pmatrix}_h \right] \tag{66}$$

but some may be singular or give unstable solutions.

As an optimization criterion can be used

$$J = \min_h \left\| ([\mathbf{n}_1 ... \mathbf{n}_m] - [\mathbf{n}_{1f} ... \mathbf{n}_{mf}])_h \right\|_F^2 \tag{67}$$

where $\boldsymbol{n}_j$, $j = 1,2, \ldots,m$ are the right eigenvectors associated with desired eigenvalues.

The procedure outlined above is extended to the state feedback control, but reconfiguration flexibility of the state control is limited, since all eigenvalues of the nominal closed-loop system have to be preserved in both control structures.

5. ILLUSTRATIVE EXAMPLE

The system model was given by (1), (2), where

$$\boldsymbol{F} = \begin{bmatrix} 0.9895 & 0.0325 & 0.5650 & 0.0207 & -0.4258 \\ 0.8714 & 0.9711 & -0.0844 & -0.0111 & 0.0312 \\ 0.5164 & 0.0101 & 0.9997 & 0.3905 & -0.0962 \\ 0.1268 & 0.0464 & -0.0017 & 0.5643 & -0.3288 \\ 0.9421 & -0.1117 & 0.0053 & 0.3431 & 0.6177 \end{bmatrix}$$

$$\boldsymbol{G} = \begin{bmatrix} -0.1638 & 0.0056 & 0.0610 \\ -0.0549 & 0.4929 & 0.0026 \\ 0.4444 & 0.0015 & -0.1765 \\ 1.5728 & 0.0101 & 0.6431 \\ 1.0863 & -0.0307 & -0.1966 \end{bmatrix}, \quad \boldsymbol{E} = \begin{bmatrix} 2.9345 \\ 1.9764 \\ 3.9234 \\ 2.5675 \\ 3.7597 \end{bmatrix}$$

$$\boldsymbol{C} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}, \quad \boldsymbol{z} = \mathrm{eig}(\boldsymbol{P}_{k1} - \boldsymbol{J}_{k1}\boldsymbol{C}_{k1}) = \begin{bmatrix} 0.1 \\ 0.2 \\ 0.3 \\ 0.4 \\ 0.5 \end{bmatrix}$$

Using the same desired eigenvalues for all estimators, for example the parameters of the first estimator was computed as follows

$$\boldsymbol{P}_1 = \begin{bmatrix} -0.2460 & 0.5704 & 0.6605 & 0.0174 & -0.4645 \\ 0.0059 & 0.2055 & 0.0535 & -0.0020 & 0.0008 \\ -0.0019 & 0.0735 & 0.4882 & 0.0006 & -0.0002 \\ -0.3021 & 0.1717 & -0.5525 & 0.3397 & -0.2777 \\ 0.4686 & -0.5990 & -0.7200 & -0.0384 & 0.7127 \end{bmatrix}$$

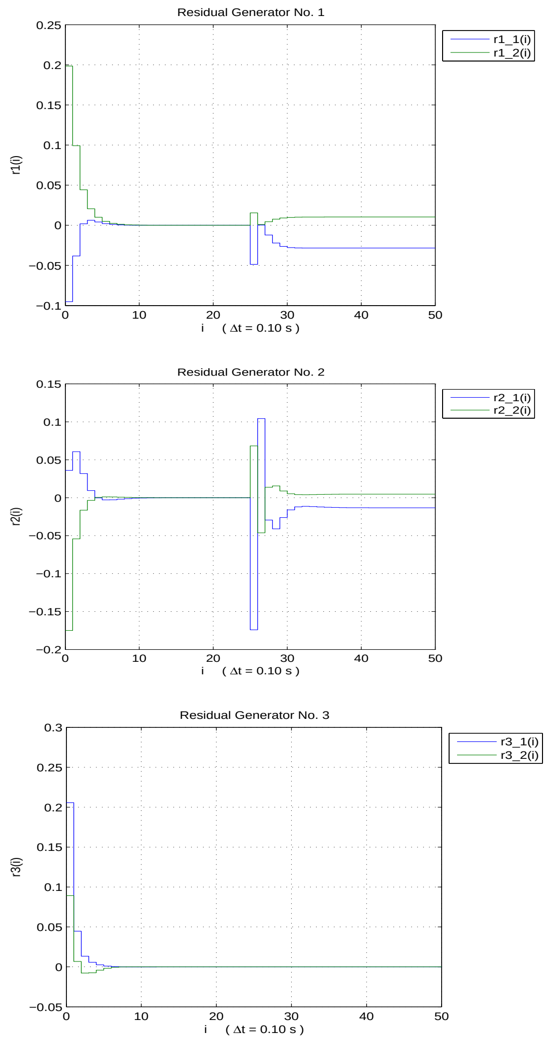

Fig. 1. Structured residual outputs for 3rd sensor fault

$$\boldsymbol{J}_1 = \begin{bmatrix} -1.8876 & -0.0263 \\ -1.1961 & -0.0592 \\ -0.0765 & -0.4864 \\ -0.2756 & -0.0160 \\ 0.5113 & -0.5113 \end{bmatrix}, \quad \boldsymbol{O}_1 = \begin{bmatrix} 1.3894 & 0.0481 \\ 0.9903 & 0.0049 \\ 0.0031 & 0.9984 \\ 0.1488 & 0.5795 \\ -0.0349 & 0.9759 \end{bmatrix}$$

$$\boldsymbol{Q}_1 = \begin{bmatrix} 1 & -1.3894 & -0.0481 & & \\ & 0.0097 & -0.0049 & & \\ & -0.0031 & 0.0016 & & \\ & -0.1488 & -0.5795 & 1 & \\ & 0.0349 & -0.9759 & & 1 \end{bmatrix}$$

$$\boldsymbol{K}_1 = \begin{bmatrix} 0.2439 & 0.2072 \\ 0.2115 & 0.0543 \\ 0.0718 & 0.4879 \\ -0.1911 & -0.6394 \\ 0.0251 & -0.0260 \end{bmatrix}, \quad \boldsymbol{T}_1 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Figure 1 shows residual generator outputs for the 3rd sensor fault. Starting transient part of the residuals responses is relevant to different initial state vectors setting in estimators and in the system.

6. CONCLUSION

Most of real systems offer the possibility to include complex control algorithms, reconfigurable control structures, fault-diagnosis methods as well as condition monitoring. The contribution gives a basic overview of these structures with hope to inspire some new points of view on this modern trend.

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