Analysis of distorted waveforms in power converter systems

Abstract. We present a method of determining the frequency, developed and based on the Prony model. The method was derived for those cases in which the process contains two dominant harmonics and noise. Additionally, in order to improve the accuracy of the method, median filtering was applied. The presented methods can be useful for practical applications in power systems, like the control systems of frequency converters.


Keywords: Prony model; frequency estimation; waveform distortion; median filter.

Introduction

Modern solid-state frequency converters used to power among others, the asynchronous motors, are characterized by distorted output voltages waveforms. Due to the control of the rotational speed of the motors, these voltage waveforms may be non-stationary in long intervals of time. Determination of the frequency of fundamental frequency component of converter output voltage is essential for their control systems and for power system apparatus diagnostics and reliability analysis [14]. Non-stationary waveforms causes that the analysis window of digital sampling methods can be very long. When applying short-time Fourier Transform (STFT), accurate results are obtained only when the sampling window has a length equal to the integer multiple of the period of the analyzed component. The length of this period is generally unknown.

Many digital methods of estimation of parameters of the basic component in real time has been developed. The best known use the spectral analysis [1,2,3], Kalman filtering [2,4], parametric methods [11, 12] or artificial neural networks [5]. Most of them have acceptable accuracy when the investigated waveform is slightly distorted. A method based on digital filtering and Prony-model estimation designed for real-time analysis of distorted signal was described in [6, 13].

The paper presents a new method of determining the frequency, developed and based on the Prony model. The method was derived for those cases in which the process contains two dominant harmonics and noise. Additionally, in order to improve the accuracy of the method, median filtering was applied. Extensive simulation studies were carried out for the proposed algorithm. Using the method of band-pass sampling of the signal [7], the reduction of sampling frequency and measurement based on a short sampling window were made possible. The results confirm the accuracy and performance of the method for distorted waveforms in the conditions of dynamic changes in the frequency of the fundamental component.

Description of the method

For N given samples of the signal it is assumed that the signal can be approximated by two sinusoidal components

\[ \hat{x}_n = A_1 \cos \left( 2\pi f_1 n T_p + \psi_1 \right) + A_2 \cos \left( 2\pi f_2 n T_p + \psi_2 \right) \]

where: \( A_1, A_2 \) amplitudes of sinusoidal components, \( f_1, f_2 \) frequencies of the components, \( \psi_1, \psi_2 \) initial phases of the components \( T_p \) - sampling period

Function (1) can be expressed as the sum of complex exponential functions

\[ \hat{x}_n = b_1 z_1^n + b_2 z_2^n \]

where:

\[ z_1 = e^{j2\pi f_1 n T_p}, \quad b_1 = A_1 e^{j\psi_1} \]

\[ z_2 = e^{j2\pi f_2 n T_p}, \quad b_2 = A_2 e^{j\psi_2} \]

It is assumed that \( z_k \) are roots of a polynomial:

\[ a_k z^n + a_k z^3 + a_k z^2 - a_k z + a_0 = 0 \] or

\[ a_0 (z - z_1) (z - z_1^*) (z - z_2) (z - z_2^*) = 0 \]

Because of the relations:

\[ 2 \cos (2\pi f_1 T_p) = z_1 + z_1^* \]

\[ 2 \cos (2\pi f_2 T_p) = z_2 + z_2^* \]

we obtained the relation between \( f_1, f_2 \) and coefficients \( a_0, a_1, \ldots, a_6 \):

\[ 2 \cos (2\pi f_1 T_p) = \frac{a_0}{2a_0} - \sqrt{\Delta} \]

\[ 2 \cos (2\pi f_2 T_p) = \frac{a_1}{2a_0} + \sqrt{\Delta} \]

where \( \Delta = \left( \frac{a_0}{a_1} \right)^2 - \frac{4}{a_0} + 8 \)

On the other hand, the coefficients \( a_0, a_1 \) can be obtained from signal samples \( x_n \) by minimization of mean square error, as follows:

\[ E = \sum_{n=1}^{N} e_n^2 \]

\[ e_n = x_n - \hat{x}_n \]
This difficult nonlinear estimation problem can be solved by using the Prony method \[8, 9\], where the approximation error (12) is expressed by the equation:

\[
\varepsilon_n = a_0 \left( x_{n-2} + x_{n+2} \right) + a_1 \left( x_{n-1} + x_{n+1} \right) + a_2 x_n
\]

We derive here the minimization of the error (13) with respect to coefficients \(a_0, a_1\):

\[
\frac{\partial E}{\partial a_0} = \frac{\partial}{\partial a_0} \left\{ \sum_{n=2}^{N-2} \left[ a_0 \left( x_{n-2} + x_{n+2} \right) + a_1 \left( x_{n-1} + x_{n+1} \right) + x_n \right]^2 \right\} = 0
\]

\[
\frac{\partial E}{\partial a_1} = \frac{\partial}{\partial a_1} \left\{ \sum_{n=2}^{N-2} \left[ a_0 \left( x_{n-2} + x_{n+2} \right) + a_1 \left( x_{n-1} + x_{n+1} \right) + x_n \right]^2 \right\} = 0
\]

leads to equations:

\[
a_0 \sum_{n=2}^{N-2} \left( x_{n-2} + x_{n+2} \right)^2 + a_1 \sum_{n=2}^{N-2} \left( x_{n-1} + x_{n+1} \right) \left( x_{n-2} + x_{n+2} \right) = -\sum_{n=2}^{N-2} x_n \left( x_{n-2} + x_{n+2} \right)
\]

\[
a_0 \sum_{n=2}^{N-2} \left( x_{n-1} + x_{n+1} \right)^2 + a_0 \sum_{n=2}^{N-2} \left( x_{n-1} + x_{n+1} \right) \left( x_{n-2} + x_{n+2} \right) = -\sum_{n=2}^{N-2} x_n \left( x_{n-1} + x_{n+1} \right)
\]

whose solution is the expression describing the factors \(a_0, a_1\):

\[
a_0 = \frac{\sum x_n \left( x_{n-1} + x_{n+1} \right) \sum \left( x_{n-1} + x_{n+1} \right) \left( x_{n-2} + x_{n+2} \right) - \sum x_n \left( x_{n-2} + x_{n+2} \right) \sum \left( x_{n-1} + x_{n+1} \right)^2}{\sum \left( x_{n-1} + x_{n+1} \right)^2 \sum \left( x_{n-2} + x_{n+2} \right)^2 - \left[ \sum \left( x_{n-1} + x_{n+1} \right) \left( x_{n-2} + x_{n+2} \right) \right]^2}
\]

\[
a_1 = \frac{\sum x_n \left( x_{n-2} + x_{n+2} \right) \sum \left( x_{n-1} + x_{n+1} \right) \left( x_{n-2} + x_{n+2} \right) - \sum x_n \left( x_{n-1} + x_{n+1} \right) \sum \left( x_{n-2} + x_{n+2} \right)^2}{\sum \left( x_{n-1} + x_{n+1} \right)^2 \sum \left( x_{n-2} + x_{n+2} \right)^2 - \left[ \sum \left( x_{n-1} + x_{n+1} \right) \left( x_{n-2} + x_{n+2} \right) \right]^2}
\]

Using (9) and (10) we obtained the frequencies of the signal components \(f_1, f_2\):

\[
f_1 = \frac{1}{2\pi T_p} \arccos \left( \frac{a_1}{4a_0} \frac{\Delta}{2} \right)
\]

\[
f_2 = \frac{1}{2\pi T_p} \arccos \left( \frac{a_1}{4a_0} + \frac{\Delta}{2} \right)
\]

**Signal Sampling**

For the discretization of signals containing two major spectral components, one can use band-pass sampling technique, which allows the sampling rate below the Nyquist frequency \([7]\). Band-pass sampling reduces the requirements for speed and memory capacity of A/D conversion. This technique consists in choosing the sampling frequency such that during the duplication of analog spectrum as a result of sampling, no distortion occurs in the output spectrum of the analyzed component.

Assuming that the frequency \(f_2\) is the carrier frequency component, with high frequency, and the useful signal bandwidth is \(2f_1\), the sampling frequency \(f_s\) should be chosen from the range obtained by the equation:

\[
\frac{2f_2 - 2f_1}{m} \geq f_s \geq \frac{2f_2 + 2f_1}{m+1}
\]

where \(m\) is any even integer number, such that \(f_s \geq 4f_1\)

Optimal sampling frequency \(f_{po}\) lies in the middle of the acceptable frequency band, according to (23):

\[
f_{po} = \frac{f_s - f_1 + f_2 + f_1}{m + 1}
\]
Investigations

In order to examine the performance of the method, a number of simulations were performed for the waveforms consisting of two sinusoidal components and Gaussian noise.

\[
x_n = A_1 \cos(2\pi f_1 n T_p + \psi_1) + A_2 \cos(2\pi f_2 n T_p + \psi_2) + e(n)
\]

where \( e(t) \) is Gaussian random noise with zero mean.

We present here the results of calculations assuming the variable frequency of the fundamental component \( f_1 \) changing in the range from 10 Hz to 100 Hz in discrete and continuous way. The high frequency component has a value \( f_2 = 10 \text{kHz} \).

During the simulation we changed the level of noise in the signal, and the parameters of the method, i.e. the width of the analysis window \( N \) and the width of median filter. Sampling frequency chosen on the basis of relation (23). The allowed sampling frequency bands \((f_{p_{max}} - f_{p_{min}})\) are shown in the Table 1.

The simulation results are presented below. Figure 2 shows a fragment of the test waveform.

Fundamental frequency component changes step-wise in time and subsequently adopts the values, \( f_1 = 10, 30, 50, 70 \) and 90 Hz respectively. The frequency of the second component is fixed at \( f_2 = 10 \text{kHz} \). The signal is sampled according to the technique of band-pass sampling with the frequency \( f_p = 470 \text{Hz} \). Amplitudes of the components are respectively \( A_1 = 1, A_2 = 0.3 \). Generated Gaussian random noise has a mean zero value and variance 0.01.

Figure 3 presents the results of measuring the frequency of primary component. The measuring window width is \( N = 1916 \) samples and a median filter was applied with a width of 3\( N \). Subsequent measurements are performed by time shift of the measuring window by one new sample. Fig. 4 is an enlarged part of Figure 3, shown to demonstrate the accuracy of determining the frequency. The accuracy depends largely on the level of noise and the width of the measuring window.

Figure 5 shows the test process where fundamental frequency component varies linearly over time and takes successively the values of \( f_1 \) from 10 to 90 Hz. The frequency of the second component is fixed at \( f_2 = 10 \text{kHz} \). The signal was sampled according to the technique of band-pass sampling with the frequency \( f_p = 470 \text{Hz} \).
The frequency determination process, as shown in above examples, because of the band-pass sampling method, allows only to correctly calculate the fundamental frequency component, whose knowledge is essential for the control system. In the case, however, when the sampling frequency is greater than Nyquist frequency of 2f1, the method correctly finds the two components of the signal.

**Fig.6. The results of measuring the frequency of fundamental component of the test process of Figure 5, N = 16, filter length 3N.**

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**Fig.7. Results of the frequency measurement of fundamental component - enlarged fragment of Figure 6, N = 16, filter length 3N.**

**Conclusions**

The paper presents a method of determining the frequency of time-varying distorted waveform, such as voltage waveforms encountered in inverter power drives.

Conducted extensive studies of the proposed method, confirm its correctness and indicate the field of possible applications.

Distorted signals were investigated, consisting of two dominant harmonics. The signal also contained noise and model of signal distortion typical for digital signal processing channel. As a result of these studies we found that with proper choice of parameters of the method, it allows the precise calculation of the frequency of fundamental component.

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**REFERENCES**


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