EXPANSION OF LOCAL GEODETIC POINT FIELD AND ITS QUALITY

ROZŠÍRENIE LOKÁLNEHO GEODETICKÉHO BODOVÉHO POĽA A JEHO KVALITA

Janka SABOVÁ [1], *Katarína PUKANSKÁ* [2]

[1] *prof. Dr. Ing., Institute of Geodesy, Cartography and GIS, BERG Faculty, Technical University of Košice, Letná 9, 042 00 Košice, Slovak Republic*
e-mail: janka.sabova@tuke.sk

[2] *Ing., PhD., Institute of Geodesy, Cartography and GIS, BERG Faculty, Technical University of Košice, Letná 9, 042 00 Košice, Slovak Republic*
e-mail: katarina.pukanska@tuke.sk

Abstract

In the development of geodetic solutions and fulfilment of the requirements and needs for completion of construction tasks and achievement of aims, we are often faced with the requirement for expansion of space or higher concentration of points in a particular local point field. Emphasis is placed on the necessity of appropriate completion of the area in question with new geodetic points of the required quality, i.e. with the acceptable 3D accuracy in terms of their comprehensive use for construction and control purposes. The present paper gives a brief outline of establishing new additional geodetic points and ensuring their quality to be compatible with the quality of the existing points in this area so that the use of both groups of points in the respective local point field was homogenous.

Abstrakt

Při vývoji geodetického řešení a plnění požadavků a potřeb na dokončení stavebních úkolů a dosažení cílů jsme často konfrontováni s požadavkem na rozšíření prostoru či vyšší koncentraci bodů v určitém místní bodovém poli. Důraz je kladen na nutnost vhodného doplnění dané oblasti novými geodetickými body v potřebné kvalitě, tj. s přijatelnou trojrozměrnou přesností z hlediska jejich komplexního využití pro stavební a kontrolní účely. Tato práce obsahuje stručný přehled o zřizování nových dodatečných geodetických bodů a zajištění jejich kvality, která by byla kompatibilní s kvalitou stávajících bodů v této oblasti, aby bylo používání obou skupin bodů v příslušném místním bodovém poli homogenní.

Key words: local geodetic network, accuracy of coordinates, covariance matrix, Gauss-Markov model

1 INTRODUCTION

The purpose of geodetic tasks is often to address the requirements related to integrating new or expanding the existing local geodetic areas in order to satisfy different building requirements. Addition and expansion of geodetic point areas is therefore a frequent requirement for carrying out construction works and providing functions of the structures and spaces being built.

The present paper, dealing with geodetic support of building activities, places particular emphasis on the necessity to appropriately complete the space in question with new geodetic points of the required quality, i.e. with the acceptable 3D accuracy in terms of their comprehensive use for construction and control purposes.

This article gives a brief overview of establishing new additional geodetic points and ensuring their quality so that they could provide all the necessary measurements together with the "old" point system to meet necessary requirements for quality assurance.

2 SATELLITE (GNSS) DETERMINATION OF NEW POINTS

Suppose we have a spatial area with the existing geodetic points B1R, B2R, B3R (Fig.1) whose 3D coordinates (X,Y,Z) were determined using GNSS technology in the ETRS89 coordinate system and whose 3D accuracy (standard deviations sx, sy, sz) is known.

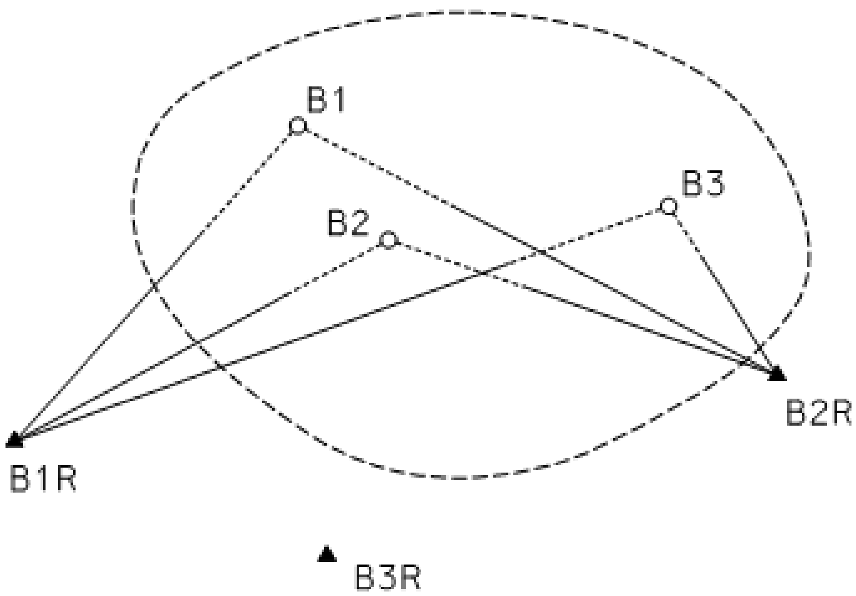

Fig.1 Reference points: B1R, B2R, B3R and new points determined: B1, B2, B3

The GNSS determination of 3D position of new points B1, B2, B3 in a local space with reference points B1R, B2R, B3R is normally carried out by the positioning of receivers in respective points using appropriate techniques and suitable methods and procedures for determining (X, Y, Z) – coordinates of the new points B1, B2, B3 resulting from the assessment of the performed measurements.

3 PROCESSING OF MEASUREMENTS USING DIFFERENT MODELS, DETERMINATION OF THE COORDINATES OF NEW POINTS AND THEIR 3D ACCURACY

For the results obtained from the measurements and various options of processing them for specific applications, i.e. precise determination of 3D coordinates of the new points B1, B2, B3, many kinds of processing techniques according to the Gauss-Markov model (GMM) or its modifications can be used such as:

GMM	with full rank of matrix **A** and massive binding,
GMM	with incomplete rank of matrix **A** (loose binding and adjustment),
GHM	Gauss-Helmert model,
GMMB	Gauss-Markov model with various conditions,
GMM	of polynomial type

and other processing models [1], [2], [3], [4], [5] as well as the works of other authors dealing with the processing of GNSS measurements and estimation of necessary parameters or the determination of adjusted values of coordinates $\hat{C} = (X, Y, Z)$ of the points B1, B2, B3 and their quality expressed by different coefficients. The information about adjustments and transformations between planar and spatial systems and about the principle of forming geodetic bases are stated in [6], [7].

Volume LVIII (2012), No.3
p. 47-51, ISSN 1802-5420

Out of all the various models, we considered using the GMM model with full rank of matrix **A** and the structure of matrix components determining the coordinates $\hat{C}$ of the points B1, B2, B3 (as shown in Fig1):

$$\underset{(9,1)}{\hat{C}} = \underset{(9,1)}{C^o} + \underset{(9,1)}{d\hat{C}}, \quad (1)$$

$$\underset{(9,1)}{d\hat{C}} = \left(\underset{(9,18)}{A^T} \underset{(18,18)}{Q_L^{-1}} \underset{(18,9)}{A} \right)^{-1} \underset{(9,18)}{A^T} \underset{(18,18)}{Q_L^{-1}} \left(\underset{(18,1)}{L} - \underset{(18,1)}{L^0} \right), \quad (2)$$

where:

$\hat{C}$ - MLS estimates of the coordinates $(\hat{X}, \hat{Y}, \hat{Z})$ of the determined points B1, B2, B3,

A - matrix of the geometric location of the given and determined points,

L - vector of the observed quantities, lengths between the points BR and B,

L^0 - vector of approximate values of quantities L,

Q_L - matrix of cofactors,

dL - vector of differences $L - L^0$ between the measured L and approximate values L^0 of the respective quantities,

C^0 - vector of approximate values of the point coordinates,

$d\hat{C}$ - estimates of the coordinate additions to values C^0.

In the corresponding adjustment process there are the following relations:

$$\underset{(9,1)}{d\hat{C}} = \underset{(9,1)}{\hat{C}} - \underset{(9,1)}{C^O} = \underset{((9,9))}{N^{-1}} \underset{(9,18)}{A^T} \underset{(18,18)}{Q_L^{-1}} \underset{(18,1)}{dL}, \quad (3)$$

$$\underset{(9,9)}{N^{-1}} = \left(A^T Q_L^{-1} A \right)^{-1}, \quad (4)$$

$$\underset{(9,18)}{H} = N^{-1} A^T Q_L^{-1} \quad (5)$$

and for the coordinate estimates $(\hat{X}, \hat{Y}, \hat{Z})$ of the individual determined points B1, B2, B3 with their numerical or stochastic properties, the valid relations are as follows:

$$\underset{(9,1)}{\hat{C}} = C^0 + H\, dL = \begin{bmatrix} \begin{pmatrix} \hat{X} \\ \hat{Y} \\ \hat{Z} \end{pmatrix}_{B1} \\ \begin{pmatrix} \hat{X} \\ \hat{Y} \\ \hat{Z} \end{pmatrix}_{B2} \\ \begin{pmatrix} \hat{X} \\ \hat{Y} \\ \hat{Z} \end{pmatrix}_{B3} \end{bmatrix}. \quad (6)$$

4 QUALITY (ACCURACY) OF THE OBTAINED COORDINATES (3D POSITIONS) OF THE NEWLY ESTABLISHED POINTS B1, B2, B3

The coordinate estimates $\left(\hat{X},\hat{Y},\hat{Z}\right)$ of the determined points B1, B2, B3 are expressed by three-dimensional location and accuracy, i.e. by standard deviations $\left(s_{\hat{x}},s_{\hat{y}},s_{\hat{z}}\right)$ of their positions expressed in the axis directions X, Y, Z of the ETRS 89 system by a corresponding covariance matrix $\sum_{\hat{C}}$.

The covariance matrix of the coordinate estimates $\sum_{\hat{C}}(9,9)$ is a structure with sub-matrices $\Sigma_{\hat{C}11},\Sigma_{\hat{C}22},\Sigma_{\hat{C}33}$ in the form as follows:

$$\underset{(9,9)}{\sum_{\hat{C}}} = s_o^2 Q_{\hat{C}} = \begin{bmatrix} \underset{(3,3)}{\sum_{\hat{C}11}} & \underset{(3,3)}{\sum_{\hat{C}12}} & \underset{(3,3)}{\sum_{\hat{C}13}} \\ & \underset{(3,3)}{\sum_{\hat{C}22}} & \underset{(3,3)}{\sum_{\hat{C}23}} \\ & & \underset{(3,3)}{\sum_{\hat{C}33}} \end{bmatrix}, \tag{7}$$

with a-posteriori variance factors

$$s_o^2 = \frac{V^T Q^{-1} V}{n-u+d-m}, \tag{8}$$

where: n – a number of measured quantities in LGS, u – a number of coordinates of the determined points, d – date defect of the network , m – the dimension of the network.

The structure of the covariance matrix $\sum_{\hat{C}}$ of the coordinate estimates (in this case for 3 determined points B1, B2, B3 in the network) with the respective estimates and their covariance characteristics (standard deviations of the determined coordinates) is as follows:

$$\underset{(3,3)}{\sum_{\hat{C}}} = \begin{bmatrix} \sum_{\hat{C}1} & \sum_{\hat{C}12} & \sum_{\hat{C}13} \\ \sum_{\hat{C}21} & \sum_{\hat{C}2} & \sum_{\hat{C}23} \\ \sum_{\hat{C}31} & \sum_{\hat{C}32} & \sum_{\hat{C}3} \end{bmatrix}, \tag{9}$$

$$\underset{(9,9)}{\sum_{\hat{C}}} = \left[\begin{array}{c|ccc|ccc|ccc} B1 & s_{\hat{x}1}^2 & s_{\hat{x}1\hat{y}1} & s_{\hat{x}1\hat{z}1} & s_{\hat{x}1\hat{x}2} & s_{\hat{x}1\hat{y}2} & s_{\hat{x}1\hat{z}2} & s_{\hat{x}1\hat{x}3} & s_{\hat{x}1\hat{y}3} & s_{\hat{x}1\hat{z}3} \\ & & s_{\hat{y}1}^2 & s_{\hat{y}1\hat{z}1} & s_{\hat{y}1\hat{x}2} & s_{\hat{y}1\hat{y}2} & s_{\hat{y}1\hat{z}2} & s_{\hat{y}1\hat{x}3} & s_{\hat{y}1\hat{y}3} & s_{\hat{y}1\hat{z}3} \\ & & & s_{\hat{z}1}^2 & s_{\hat{z}1\hat{x}2} & s_{\hat{z}1\hat{y}2} & s_{\hat{z}1\hat{z}2} & s_{\hat{z}1\hat{x}3} & s_{\hat{z}1\hat{y}3} & s_{\hat{z}1\hat{z}3} \\ \hline B2 & & & & s_{\hat{x}2}^2 & s_{\hat{x}2\hat{y}2} & s_{\hat{x}2\hat{z}2} & s_{\hat{x}2\hat{x}3} & s_{\hat{x}2\hat{y}3} & s_{\hat{x}2\hat{z}3} \\ & & & & & s_{\hat{y}2}^2 & s_{\hat{y}2\hat{z}2} & s_{\hat{y}2\hat{x}3} & s_{\hat{y}2\hat{y}3} & s_{\hat{y}2\hat{z}3} \\ & & & & & & s_{\hat{z}2}^2 & s_{\hat{z}2\hat{x}3} & s_{\hat{z}2\hat{y}3} & s_{\hat{z}2\hat{z}3} \\ \hline B3 & & & & & & & s_{\hat{x}3}^2 & s_{\hat{x}3\hat{y}3} & s_{\hat{x}3\hat{z}3} \\ & & & & & & & & s_{\hat{y}3}^2 & s_{\hat{y}3\hat{z}3} \\ & & & & & & & & & s_{\hat{z}3}^2 \end{array}\right], \tag{10}$$

where the covariance matrix $\sum_{\hat{C}}$ with the corresponding sub-matrices $\sum_{\hat{C}j}\ (j=1,2,...9)$ is the aposterioiri (empirical) matrix n – dimensional random vector L (dL), i.e. the matrix of standard deviations in the coordinate estimates $\left(\hat{X},\hat{Y},\hat{Z}\right)$ of the points B1, B2, B3 in the respective local geodetic network (LGS).

Volume LVIII (2012), No.3
p. 47-51, ISSN 1802-5420

5. CONCLUSION

The expansion of space or higher concentrations of points in a particular local point field are frequent tasks in geodetic solutions and fulfilment of requirements and needs of construction tasks and aims. They are related to different requirements for the quality of newly established points ensuring that the quality of new points is compatible with that of the existing points in the given area and that the use of both groups of points in the respective local point field is homogeneous.

REFERENCES

[1] HOFFMANN-WELLENHOF,B. et al.(2001): *Global Positioning System, Theory and Praxis*, Springer, Wien.

[2] KOCH,K.R.(2008): *Parameterschätzung und Hypothesen tests in linearen Modellen*, Dümmler, Bonn.

[3] LEICK,A.(2007): *GPS Satellite Surveying*, J.Wiley, NewYork.

[4] NIEMEIER,W.(2002): *Ausgleichungs-rechnung*, W.de Gruyter, NewYork.

[5] SEEBER,G.(2003): *Satellite geodesy*: Foundations, Methods and Applications. De Gruyter, Berlin-New York, 589 pp.

[6] STAŇKOVÁ,H; ČERNOTA,P.: *A Principle of Forming and Developing Geodetic Bases in the Czech Republic, Geodesy and Cartography*, Vilnius: Technica, 2010, Vol.36, No.3 p. 103-112, ISSN 1392-1541 print / ISSN 1648-3502 online.

[7] STAŇKOVÁ,H., ČERNOTA,P.: *Spatial transformation between S-SK and ETRS89 systems*. In: Ostrava, VŠB-TU Ostrava (ed.): VŠB – Technical University of Ostrava Annual Proceedings of Science and Technology at VŠB-TUO, Czech Rep., IV(2010), pp.174-179, Ostrava, Czech Rep., 2010.

RESUMÉ

Pre geodetické úlohy sú časté požiadavky doplňovania resp. rozšírenia lokálnych geodetických oblastí za účelom zabezpečenia rovnakých výstavbových požiadaviek. Doplňovanie a rozširovanie geodetických bodových oblastí je preto frekventovanou požiadavkou pre realizáciu rôznych výstavbových cieľov a zabezpečenia funkcií príslušných budovaných objektov a priestorov.

V danom pojednaní o geodetickom zabezpečení výstavbových úloh sa zdôrazňuje potrebné a vhodné dobudovanie príslušného priestoru novými geodetickými bodmi s potrebnou - požadovanou kvalitou, t.j. s prijateľnou 3D - presnosťou z hľadiska ich všestranného využitia pre výstavbové a kontrolné ciele.