Quantitative evaluation of the synergistic effects of failures in a critical infrastructure system

David Rehak*, Jiri Markucia, Martin Hromada, Karla Barcova

Faculty of Safety Engineering, VSB – Technical University of Ostrava, Lumiřová 13, 70030 Ostrava-Vyskovice, Czech Republic
Faculty of Applied Informatics, Tomas Bata University in Zlín, Nad Stráněmi 4511, 76005 Zlín, Czech Republic

A critical infrastructure is a complicated system whose failure (in whole or in part) has a significant impact on national interests, including security, the economy and basic human needs. The system consists of relevant sectors, elements and their mutual linkages. In order to study critical infrastructures, it is necessary to apply a systems approach based on cross-sectoral evaluation and research into the linkages between the individual critical infrastructure sectors. Specifically, it is necessary to describe the individual vertical and horizontal levels of each critical infrastructure and the associated linkages. From this point-of-view, a critical infrastructure is embedded within the broader context of emergencies and enterprises, representing a compact and mutually-interconnected system.

This paper focuses on quantitatively assessing the impacts of critical infrastructure failures. It presents a theory of synergistic linkages, their levels and the synergistic effects due to the joint action of impacts, which increase the overall impact on the critical infrastructure and on society. The concepts are formalized in the SYNEFIA methodology, which is applied in a case study involving the critical infrastructure of the Czech Republic. In particular, the methodology is applied to determine the synergistic effects of disruptions to multiple sub-sectors of the Czech infrastructure.

*Corresponding author.
E-mail address: david.rehak@vsb.cz (D. Rehak).

http://dx.doi.org/10.1016/j.ijcip.2016.06.002
1874-5482/© 2016 The Authors. Published by Elsevier B.V. This is an open access article under the CC BY-NC-ND license (http://creativecommons.org/licenses/by-nc-nd/4.0/).
severity of the failure, its cause (i.e., character of the threat) and the criticality of the affected elements or sectors. Such impacts are often expressed in terms of economic losses, number of people affected, size of the affected region and other factors that fall into three basic categories: (i) critical proportion; (ii) critical time; and (iii) critical quality [11]. When the threshold values of the impacts (i.e., sectoral and cross-cutting criteria) are exceeded, the corresponding infrastructure sectors and their elements are deemed to be critical; taken together, they constitute the critical infrastructure [10].

The importance of critical infrastructure protection was first highlighted by the United States in 1995. Over the years, critical infrastructure protection activities were initiated by other countries – Canada in 1998 and the United Kingdom, Sweden and Switzerland in 1999. Since the infamous attacks of September 11, 2001, many European countries have defined their critical infrastructure assets and launched critical infrastructure protection efforts.

In the National Infrastructure Protection Plan of 2013 [31], the U.S. Department of Homeland Security defined the critical infrastructure as “systems and assets, whether physical or virtual, so vital to the United States that the incapacity or destruction of such systems and assets would have a debilitating impact on security, national economic security, national public health or safety, or any combination of those matters.” The Australian Government [1] defines critical infrastructure as “those physical facilities, supply chains, information technologies and communication networks which, if destroyed, degraded or rendered unavailable for an extended period, would significantly impact on the social or economic wellbeing of the nation or affect Australia’s ability to conduct national defense and ensure national security.”

At the European Union level, the term critical infrastructure is defined in two key documents. The first is the Green Paper on the Programme of Critical Infrastructure Protection [7], which was published in 2005 by the European Commission. The second is the Council Directive on the Identification and Designation of European Critical Infrastructures and on the Assessment of the Need to Increase Their Protection [10], which was published as a follow-up to the Green Paper in 2008. The council directive defines critical infrastructure as “an asset, system or part thereof located in Member States which is essential for the maintenance of vital societal functions, health, safety, security, economic or social wellbeing of people, and the disruption or destruction of which would have a significant impact in a Member State as a result of the failure to maintain those functions.” The directive leaves the responsibility for critical infrastructure protection to national authorities.

Critical infrastructures are complex. In effect, critical infrastructures and their dependencies form a system of systems [20,25]. The overall critical infrastructure has an obvious hierarchy, consisting of individual sectors such as energy and transportation, along with their linkages [25]. The sectors consist of elements that are considered to be a basic part of the system. Currently, it is possible to distinguish two basic methodological approaches for the risk assessment of critical infrastructures. The first is the sectoral approach, where each sector is assessed separately with its own risk assessment methods. The second is the systems approach, where individual critical infrastructure sectors are deemed to be interconnected networks.

Research in the critical infrastructure protection field [8,9] should improve the fidelity and precision of simulation tools for modeling the impact of critical infrastructure malfunctions [14]. The research should also be extended to the synergies and synergistic effects of infrastructure failures. Dynamic functional modeling [29] is a promising approach that can consider synergistic effects. However, it is currently used to simulate the systemic impacts on critical infrastructures (i.e., basic impacts without synergies) and does not support the modeling and simulation of synergistic effects.

2. National critical infrastructure system

The hierarchic arrangement of a national critical infrastructure system has three levels that constitute a vertical classification:

- System level.
- Sector level.
- Element level.

The system level is the basic classification of a critical infrastructure according to its functions. This level comprises: (i) the technical infrastructure and (ii) the socioeconomic infrastructure. For example, the technical infrastructure in the Czech Republic includes the energy, transport, water supply, food processing, agriculture, industry, and communications and information systems sectors. The socioeconomic infrastructure in the Czech Republic includes health care, financial and currency markets, emergency services and public administration. There are significant dependencies between the two types of critical infrastructure. For instance, all the socioeconomic sectors require the commodities produced by the technical infrastructure sectors. The technical sectors depend on the socioeconomic sectors, especially in crisis situations.

The sector level is made up of the individual sectors of a critical infrastructure (e.g., energy and water supply). This level represents the classification of actual sectors of the critical infrastructure and their linkages.

The individual components that form the element level are the basic building blocks of the system hierarchy of the sectors. The elements are relevant to the system due to the impacts produced by their failure. The elements can be classified into four categories based on their potential impacts [26]. Table 1 provides a detailed description of the classification.

In addition to the vertical categorization, it is also possible to view a critical infrastructure system with respect to its horizontal linkages. This creates a context with the surrounding processes and operators. The linkages define what impacts a critical infrastructure and what can be affected in the event of a failure. The correlation of the cause-failure-impact scenario
shown in Fig. 1 was identified on the basis of these linkages. The failure of critical infrastructure functions is caused by stimuli that have various intensity levels according to their character, scope and duration (i.e., Levels I–III emergencies). The impacts may have different levels based on the intensity levels of the causes and the resistance levels (protection) of the individual elements of a critical infrastructure system.

The easiest way of identifying critical infrastructure elements is to use a bottom-up approach. In this approach, critical elements in one sector are gradually identified, starting with the elements at the lowest level, followed by selections of critical elements at progressively higher levels. However, this purely deterministic approach is no longer acceptable due to the strong cohesion of individual critical infrastructure sectors and, especially, the

<table>
<thead>
<tr>
<th>Categorization of the critical infrastructure system elements</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Critical infrastructure category III elements (elements at the local level)</td>
<td>Disturbances of these elements impact social life in a municipality or municipal district. The disturbances lead to interruptions in the delivery of services (e.g., provision of food, electric power and potable water). A Level III Emergency (lowest level) is declared. The non-functional elements in this category are replaced by adopting special organizational measures or temporary solutions provided by emergency services (it is still possible to supply food, backup electric power and potable water).</td>
</tr>
<tr>
<td>Critical infrastructure category II elements (elements at the regional level)</td>
<td>Disturbances of these elements impact social life in several municipalities, municipal districts or an entire region. When the elements of this category are disrupted, the problem is solved by the infrastructure operators in cooperation with emergency services at the regional level. A Level II Emergency is declared and regulatory measures are adopted. Humanitarian aid is provided from national resources.</td>
</tr>
<tr>
<td>Critical infrastructure category I elements (elements at the national level)</td>
<td>Disturbances of these elements impact national security and the provision of essentials to citizens in two or more regions, or in an entire country. When the elements in this category are disrupted, the problem is solved by the infrastructure operators (based on approved crisis preparedness and emergency plans) in close cooperation with the ministries and central administration authorities responsible for the regions. The elements of this category are practically irreplaceable and their disruptions may only be resolved provisionally or through the use of resources secured in advance (e.g., water and fuel). A Level I Emergency is declared and regulatory measures are adopted. International humanitarian aid may be requested.</td>
</tr>
<tr>
<td>Special critical infrastructure elements category (elements at the international level)</td>
<td>Disturbances of these elements impact the national security of two or more states. A Level I Emergency is declared. Extensive regulatory measures are adopted and there is close international cooperation, coordination and organization of humanitarian aid (e.g., by the Emergency Response Coordination Centre [6] or the United Nations Office for the Coordination of Humanitarian Affairs [30]).</td>
</tr>
</tbody>
</table>

**Fig. 1 – Correlation between the intensity of cause, failure and impact in a critical infrastructure system.**
linkages of their elements [24,25]. A better approach is to assess the joint causes and dependencies of the individual critical infrastructure sectors (not just each element individually) and their actual impacts. Researchers have devoted considerable effort to modeling and simulating the overall impact [21,33], not only on the interests protected by the state (i.e., security, economy and basic human needs [10]), but also on the elements of individual sectors (e.g., service loss, economic loss and reputation loss).

3. Critical infrastructure system linkages

A critical infrastructure system and its organization must be viewed comprehensively, with the individual elements and sectors interlinked via various types of linkages (e.g., relations and connections). The basic structure of the linkages arises from their character. A one-way linkage represents an influence or dependency; a two-way linkage expresses an interdependency (see Fig. 2). Rinaldi et al. [25] have categorized interdependencies as physical, cyber, geographic and logical in nature, and they argue that interdependencies increase the risk of disturbances and failures in multiple interconnected infrastructures. Pederson et al. [24] have further categorized infrastructure linkages, providing lower levels of detail. Interested readers are referred to [13,15,19,27] for discussions of critical infrastructure interdependencies.

All the types of linkages discussed above exist in a critical infrastructure system – at the vertical level (area-sector-element) and at the horizontal level (cause-failure-impact). As shown in Fig. 3, the linkages occur at the following levels:

- Between elements of critical infrastructure sectors (i.e., cross-sectoral linkages).
- Between elements within a critical infrastructure sector (i.e., sectoral linkages).
- Between elements of a critical infrastructure and society.

As with any network, a critical infrastructure system has elements with different levels of importance (criticality). The damage, disruption or failure of an important (critical) element has a more or less serious impact based on the number and character of linkages that define its level of effect, dependence or interdependence. A failure may not only cause a serious disruption of a sector or an entire critical infrastructure system, but it...
can also impact national interests such as security, the economy and basic human needs [10].

4. Impacts of critical infrastructure system failures

The prediction and subsequent minimization of the impacts of failures of individual elements, sectors and entire critical infrastructures are important components of critical infrastructure protection research. Prediction involves an analysis of all the available information about the nature of the impacts, which depend on several external and internal factors of the system of interest. The external factors include the resilience of society and the character, scope and duration of the event. The internal factors include the type and scope of the system failure inside the system; interested readers are referred to Rinaldi et al. [25] for details about system linkages and system resilience. The impacts are characterized by the scope, structure, intensity, duration and effects of the adverse event (see Fig. 4).

A critical infrastructure system failure produces two types of impacts. The first type constitutes the negative impacts within the critical infrastructure system when the failure of one infrastructure sector causes a failure of another sector or its elements (i.e., cascading effect [25]). The second type corresponds to the negative impacts outside the system, specifically, on society, including national interests such as security, the economy and basic human needs [10].

In both cases, the impacts may be classified as direct or indirect from the structural point-of-view. The immediate effect of the disturbed sector on another sector or directly on society is considered to be a direct or primary action. In contrast, the indirect impacts occur implicitly through a sector of a critical infrastructure, regardless of whether or not it affects another sector or society as a whole. The indirect impacts may be secondary (through one sector) or multi-structural (through several sectors).

Other important characteristics of an impact are its intensity and duration. The impact intensity depends on the scope of a sector failure (i.e., how it affects other sectors and the levels of sector interdependencies. When the linkages are weak, the impact intensity is low and the impact on other sectors is only partial. However, when the linkages are strong, the impact intensity is high and the impact on other sectors can be devastating (or absolute). The impact duration is obviously an important variable; the duration may be short-term, medium-term or long-term. Ouyang et al. [22] discuss the typical time progression of a critical infrastructure disruption, which is divided into: (i) a prevention period; (ii) a propagation period; and (iii) a damage, assessment and restoration period.

Another important characteristic is impact effect. If the impact of a disrupted sector only influences another sector or society in one way, then the impact effect is referred to as a

![Fig. 4 – Aspects that create the character of impacts in a critical infrastructure system.](image-url)
single impact. However, if the impact effects are multi-way (e.g., a combination of direct and indirect impacts) and occur in real-time, then the impact effects are synergistic in nature.

The term synergy comes from the Greek syn-ergazomai, which means cooperation or joint action. Historically, the term has been used to describe the cooperation of several people and the theological notion of cooperation of man with God (synergism). Dictionary.com [5] describes synergy as “the interaction of elements that, when combined, produce a total effect that is greater than the sum of the individual elements, contributions, etc.” The online dictionary also defines synergy in physiology and medicine as a cooperative action of two or more muscles, nerves, or the like. In the areas of biochemistry and pharmacology, synergy is a cooperative action of two or more stimuli or drugs. In business management, synergism is the potential ability of individual organizations or groups to be more successful or productive as a result of a merger.

The term synergy is used in various forms in many areas of human activity, but it has rarely been used in connection with the critical infrastructure. The first mention of synergy in the critical infrastructure domain was in 2001 [25], but only in the context of linkages in the economic infrastructure. Nevertheless, a classic example of synergistic effects in the critical infrastructure domain as emerged – the Fukushima Daiichi nuclear disaster of March 2011. The earthquake, tsunami and nuclear cooling system failure induced massive synergistic effects that may well continue to impact Japanese society for decades.

Based on the discussion above, synergies potentially exist at all three horizontal levels of a critical infrastructure system:

- Synergy of emergencies (Level I synergy): This occurs due to the interactions of two and more emergencies on an element or sector of a critical infrastructure.

Fig. 5 – Synergistic effects in a critical infrastructure system.
Synergy of elements or sectors (Level II synergy): This occurs due to the interactions of the impacts of failures of two or more critical infrastructure elements or sectors on a third critical infrastructure element or sector.

Synergy of societal impacts (Level III synergy): This occurs as a result of the combined interactions of the impacts of failures of critical infrastructure elements or sectors with the impacts of emergencies on society. This creates an imaginary “ring of synergy” of all the current impacts on society.

Fig. 5 shows the individual synergistic effects that can occur in a critical infrastructure system. The synergistic effects are created by the synergy of impacts and the aggregated effects of the interactions of impacts. This situation is symbolically expressed as $2 + 2 > 4$ or $2 + 2 = 5$. BusinessDictionary.com defines synergistic effect as “an effect arising between two or more agents, entities, factors or substances that produces an effect greater than the sum of their individual effects.”

Fig. 6 shows a simplified example of the synergistic effects arising from concurrent failures of two critical infrastructure sectors. Nieuwenhuijs et al. [19] have specified several functions to express the impacts between times $t_1$ and $t_2$ as well as between times $t_3$ and $t_4$. In particular, Fig. 6 shows the impacts of the failures of unspecified energy sector elements and potable water sector elements. The failure of the energy sector has a social impact $C_E$ that occurs at time $t_0$ due to the effect of an emergency (e.g., storm). The failure of the potable water sector is the result of a cut-back at time $t_1$, which causes the social impact to increase to $C_E + W$. The maximum impact caused by the non-functioning of the two sectors occurs at time $t_2$.

During the time interval $[t_1, t_3]$, a synergy of impacts exists (i.e., Level III synergy) of the energy and potable water sectors on society, which may lead to synergistic effects that increase the overall impact to $C_{E+W}$. When the energy sector is restored at time $t_3$, the synergies and synergistic effects cease. The restoration of the potable water sector continues until time $t_4$. At the same time, the synergistic effects and, thus, the overall societal impact, may increase or decrease due to the concurrent effects of the non-functioning of two and more elements/sectors in the critical infrastructure.

The opposite of a synergistic effect is an antagonistic effect, which is symbolically expressed as $2 + 2 < 4$ or $2 + 2 = 3$. In physiology and medicine, this effect is described...
as “the opposing action of substances, like drugs, that – when taken together – decrease the effectiveness of at least one of them” [5]. In the case of a major event or disaster, the societal and psychological impacts may cause cascading impacts in the critical infrastructure, and common cause disruptions may be relegated to minor importance.

Prediction of the synergistic effects in a critical infrastructure system is much more difficult than predicting impacts. While the variables involved in predicting single impacts are relatively obvious, the variables that determine the synergistic effects vary considerably and their specification is limited by the number and intensity of the emergencies, number of affected critical infrastructure sectors, resilience of the affected critical infrastructure sectors and societal vulnerabilities. Interested readers are referred to [18] for more information about this subject.

5. Synergistic effect impact quantification

This section discusses the application of the SYNEFIA methodology to quantify the synergistic impacts arising from a critical infrastructure failure. The theoretical basis of the methodology is the notion of symmetric operational impacts – specifically, that the degree of an impact on society is directly proportional to the significance of a sector, sub-sector or element of the critical infrastructure system.

This section applies the SYNEFIA methodology at the sub-sector level. In this case, the sub-sector significance is directly proportional to the activity and passivity of the sub-sector in the critical infrastructure system.

The SYNEFIA methodology has five phases:

2. Critical infrastructure sub-sector correlation analysis.
3. Critical infrastructure sub-sector significance determination.
4. Impact evaluation.
5. Synergistic effect determination.

5.1. Critical infrastructure sub-sector identification

The identification phase creates an inventory of all the critical infrastructure sub-sectors that must be evaluated in the geographical region of interest (Fig. 1). This phase also determines the maximum number of sub-sectors to be considered in the geographical region of interest. The inventory is specific to each national critical infrastructure system. Indeed, the definition and designation of critical infrastructure sectors differs considerably for different countries [28]. For example, the Annex to the European Council Directive of 2008 [10] defines sub-sectors only for the energy and transport sectors. Consequently, an inventory of sub-sectors may have to be created for a geographical region, country or group of countries.

5.2. Critical infrastructure sub-sector correlation analysis

The second phase evaluates the interactions between the sub-sectors of a critical infrastructure system. This research has employed the KARS method [23], which is primarily used for quantitative risk analysis based on risk correlation.

The first step is to determine the correlations of the sub-sectors. The correlations are evaluated using pairwise comparisons that compare the importance of the two selected sub-sectors. The more important sub-sector from each pair is selected. Table 2 shows the correlation results.

The second step is to determine the activities and passivities that are latent in each critical infrastructure sub-sector. The activity coefficient $K_A S_i$ of sub-sector $S_i$ expresses the full potential of sub-sector $S_i$ to cause failures of other sub-sectors. It is computed as:

$$K_A S_i = \frac{\sum_{j=1}^{n} A_j}{n-1}$$  \hspace{1cm} (1)

where $A_i$ is the sum of the activity of sub-sector $S_i$ and $n$ is the number of sub-sectors.

The passivity coefficient $K_P S_i$ of sub-sector $S_i$ expresses the potential that the other sub-sectors can cause a failure of sub-sector $S_i$. It is computed as:

$$K_P S_i = \frac{\sum_{j=1}^{n} P_j}{n-1}$$  \hspace{1cm} (2)

where $P_i$ is the sum of the passivity of sub-sector $S_i$ and $n$ is the number of sub-sectors.

The passivity coefficients of the electricity and natural gas sub-sectors are determined to illustrate the computations. Using the data in Table 2 and applying Eq. (2), the passivity

<table>
<thead>
<tr>
<th>Table 2 – Correlation results.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Index</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>4</td>
</tr>
<tr>
<td>$\Sigma$</td>
</tr>
</tbody>
</table>

$x$: Sub-sector failure can be caused internally.

1: Sub-sector $S_i$ can cause the failure of sub-sector $S_j$.

0: Sub-sector $S_i$ cannot cause the failure of sub-sector $S_j$. 


5.3. Critical infrastructure sub-sector signficance determination

The significance of critical infrastructure sub-sectors is determined in two steps. In the first step, the quantification results are obtained by plotting each sub-sector on a graph based on its activity and passivity coefficients as shown in Fig. 7. The graph is then divided into four segments to classify the sub-sectors according to their significance. Segment I contains sub-sectors with the highest levels of influence and dependence (primary significance). Segment II contains sub-sectors with the highest level of dependence (secondary significance). Segment III contains sub-sectors with the highest influence (secondary significance). Segment IV contains sub-sectors with the lowest levels of influence and dependence (tertiary significance).

In order to divide the graph into four segments, it is necessary to specify the lines P1 and P2. The parameters of the two lines are set using the Pareto Principle [16], which assumes that 80% of the sub-sectors are in Segment I (most significant critical infrastructure sub-sectors). The parameter $P_1$ of line P1 is computed as:

$$P_1 = K_{A_{\text{max}}} - \frac{(K_{A_{\text{max}}} - K_{A_{\text{min}}}) \times 80}{100}$$

(3)

where $K_{A_{\text{max}}}$ is the maximum value of the activity coefficient and $K_{A_{\text{min}}}$ is the minimum value of the activity coefficient.

Similarly, the parameter $P_2$ of the line P2 is computed as:

$$P_2 = K_{P_{\text{max}}} - \frac{(K_{P_{\text{max}}} - K_{P_{\text{min}}}) \times 80}{100}$$

(4)

where $K_{P_{\text{max}}}$ is the maximum value of the passivity coefficient and $K_{P_{\text{min}}}$ is the minimum value of the passivity coefficient.

In the second step, the significance of each sub-sector is determined mathematically as the simple sum of the sub-sector activity and passivity in the critical infrastructure system. Thus, sub-sectors with the higher composite levels of influence and dependence are considered to be more significant. The significance $R_i$ of sub-sector $S_i$ is given by:

$$R_i = K_{A_i} + K_{P_i}$$

(5)

where $K_{A_i}$ is the activity coefficient of sub-sector $S_i$ and $K_{P_i}$ is the passivity coefficient of sub-sector $S_i$.

5.4. Impact evaluation

The fourth phase determines the impact of each sub-sector. The impact is computed as the ratio of the sub-sector failure impact to the overall critical infrastructure system failure impact. Note that, in this phase and in the next phase, the results are expressed as percentages. Thus, the impact (percentage) $C_i$ of sub-sector $S_i$ on society is computed as:

$$C_i = \frac{R_i}{\sum_{i=1}^{n} R_i} \times 100$$

(6)

where $R_i$ is the significance of sub-sector $S_i$ and $n$ is the number of sub-sectors.

5.5. Synergistic effect determination

The fifth and final phase of the SYNEFIA methodology is to determine the synergistic effects. In particular, the synergistic effects due to cascading failures of two or more sub-sectors are computed. The synergistic effects can be viewed as arising from the lack of resilience of a critical infrastructure with respect to the impact of an incident, causing accumulative effects that increase the impact on the system and society.

Fig. 8 presents a graphical representation of the synergistic effects due to disruptions to three sub-sectors. In the figure, an incident X causes a failure in sub-sector $S_1$ (e.g., electricity). The failure impacts society Y as well as other sub-sectors (e.g., $S_2$). The impacts are represented by $C_1$.

The failure in sub-sector $S_1$ causes a cascading effect at the same time due to the failure of sub-sector $S_2$ (e.g.,
information and communications systems). The impact of the failure in sub-sector $S_2$ on society $Y$ is expressed by $C_3$.

The synergistic effect level $C_{21}$ is caused by the simultaneous failure of sub-sectors $S_1$ and $S_2$, which occurs due to the failure of sub-sector $S_1$. The synergistic effect level $C_{23}$ is caused by the simultaneous failure of sub-sectors $S_2$ and $S_3$ (e.g., road transport), which occurs due to the failure of sub-sector $S_2$. The synergistic effect on society caused by the failures of all the affected sub-sectors is computed as the sum of all the sub-sectoral synergistic effects.

The synergistic effects are determined based on the assumption that the impact of the sub-sector failure on society is proportional to the impacts of the sub-sector failure on other critical infrastructure sub-sectors that are connected to it through their mutual dependencies (see Fig. 2). For example, if the impact of an electricity sub-sector failure is 5%, then the sub-sector failure impact on other sub-sectors (e.g., information and communications systems, and road transport) would also be 5%. This value increases the impact of the affected sub-sector on society, which expresses the synergistic effect.

Based on the above discussion, the synergistic effect (percentage) of the simultaneous impacts of sub-sectors $S_i$ and $S_j$ on society induced by sub-sector $S_i$’s influence on sub-sector $S_j$ is given by:

$$C_{ij} = \frac{C_i \times C_j}{100}$$  \hspace{1cm} (7)

where $C_i$ is the impact (percentage) of sub-sector $S_i$ on society and $C_j$ is the impact (percentage) of sub-sector $S_j$ on society, which is also the impact (percentage) of sub-sector $S_j$ on sub-sector $S_i$. Having determined the synergistic impacts, the overall impact (percentage) of a critical infrastructure system on society $C_Y$ is given by:

$$C_Y = \sum_{i=1}^{n} C_i + \sum_{i=1}^{n} C_{ij}$$  \hspace{1cm} (8)

where $C_i$ is the impact (percentage) of sub-sector $S_i$ on society and $C_{ij}$ is the synergistic effect (percentage) of the simulta-
neous impacts of sub-sectors $S_i$ and $S_j$ on society induced by sub-sector $S_i$'s influence on sub-sector $S_j$.

6. SYNEFIA methodology application

This section presents a case study involving the critical infrastructure of the Czech Republic. The SYNEFIA methodology is applied to determine the synergistic effects of disruptions to multiple sub-sectors of the Czech infrastructure.

The Czech Republic's critical infrastructure currently comprises nine sectors and 27 sub-sectors [12]. As mentioned above, the SYNEFIA methodology is applied only to the 27 sub-sectors.

The first phase of the methodology is to identify the critical sub-sectors. Table 3 presents the results of this step.

The second phase focuses on sub-sector correlation analysis. The correlations are evaluated using pairwise comparisons of sub-sectors; the more important sub-sector of each pair is selected. Table 4 presents the matrix created by the pairwise comparisons of sub-sectors.

Table 5 presents the activity and passivity coefficients of the 27 sub-sectors. The activity coefficients $K_{AS_i}$ and passivity coefficients $K_{PS_i}$ were calculated using Eqs. (1) and (2), respectively.

The third phase of the SYNEFIA methodology is to determine the significance of the individual sub-sectors. As described in the previous section, the sub-sectors are plotted on a graph based on their activity and passivity coefficients. The graph is then divided into four segments using the two lines with parameters $P_1$ and $P_2$ computed using Eqs. (3) and (4), respectively. Fig. 9 shows the resulting graph with the sub-sectors placed in four segments.

Based on the placement of the sub-sectors in the graph, the following classification of sub-sectors is obtained based on their significance:

- Primary sub-sectors (Segment I): 1, 2, 3, 4, 8, 9, 10, 19, 20, 21.
- Secondary sub-sectors (Segments II and III): 6, 7, 11, 12, 13, 14, 15, 16, 17, 18, 23, 24, 25, 26, 27.
- Tertiary sub-sectors (Segment IV): 5, 22.

The fourth phase is to determine the interrelated sub-sectors that are affected by cascading consequences. These sub-sectors are identified based on correlation analysis. Table 4 shows the results. For reasons of space, only the correlations of Sub-Sector 1 (electricity) with the primary significant sub-sectors are assessed.

Next, the impact of each sub-sector on society is determined using Eq. (6). To accomplish this, the significance of each sub-sector in the Czech Republic's critical infrastructure has to be computed using Eq. (5). Table 6 presents the results of the two computations.

The fifth and final phase of the SYNEFIA methodology is to determine the impact of each sub-sector on society. Following this, the synergistic effects of the failures of two or more sub-sectors due to cascading consequences are computed. In the case study, the synergistic effects are computed only for

### Table 4 – Correlation analysis of the first 12 critical infrastructure sub-sectors in the Czech Republic.

<table>
<thead>
<tr>
<th>Index</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Electricity</td>
<td>x</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>Natural gas</td>
<td>1</td>
<td>x</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>Petroleum/products</td>
<td>1</td>
<td>1</td>
<td>x</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>Water management</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>x</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>Crop production</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>x</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>Livestock</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>x</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>Food production</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>x</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>Healthcare</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>x</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>9</td>
<td>Road transport</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>x</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>Rail transport</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>x</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>11</td>
<td>Air transport</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>x</td>
</tr>
<tr>
<td>12</td>
<td>Inland water transport</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

x: Sub-sector failure can be caused internally.
1: Sub-sector $S_i$ can cause the failure of sub-sector $S_j$.
0: Sub-sector $S_i$ cannot cause the failure of sub-sector $S_j$. 
the cascading failures of Sub-Sector 1 (electricity), Sub-Sector 8 (health) and other key sub-sectors shown in Fig. 11.

Eq. (7) is used to determine the synergistic effects and Eq. (8) is used to compute the overall critical infrastructure system failure impact on society. Table 7 presents the results.

This case study demonstrates the utility of the SYNEFIA methodology in determining the synergistic effects of sub-sector failures on the overall critical infrastructure of a nation and on its society. In practice, however, instead of sustained sub-sector failures, it is more likely to encounter partial disruptions or failures of some elements of a critical infrastructure. In such instances, before determining the synergistic effects, it is necessary to understand and evaluate the percentage decreases in the impacts $C_i$ produced by the sub-sectors of interest.

<table>
<thead>
<tr>
<th>Sub-sector index</th>
<th>Activity coefficient ($K_A S_i$)</th>
<th>Passivity coefficient ($K_P S_i$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.73</td>
<td>0.46</td>
</tr>
<tr>
<td>2</td>
<td>0.27</td>
<td>0.35</td>
</tr>
<tr>
<td>3</td>
<td>0.58</td>
<td>0.42</td>
</tr>
<tr>
<td>4</td>
<td>0.35</td>
<td>0.50</td>
</tr>
<tr>
<td>5</td>
<td>0.15</td>
<td>0.23</td>
</tr>
<tr>
<td>6</td>
<td>0.15</td>
<td>0.35</td>
</tr>
<tr>
<td>7</td>
<td>0.15</td>
<td>0.46</td>
</tr>
<tr>
<td>8</td>
<td>0.27</td>
<td>0.73</td>
</tr>
<tr>
<td>9</td>
<td>0.35</td>
<td>0.38</td>
</tr>
<tr>
<td>10</td>
<td>0.38</td>
<td>0.54</td>
</tr>
<tr>
<td>11</td>
<td>0.08</td>
<td>0.50</td>
</tr>
<tr>
<td>12</td>
<td>0.15</td>
<td>0.35</td>
</tr>
<tr>
<td>13</td>
<td>0.73</td>
<td>0.19</td>
</tr>
<tr>
<td>14</td>
<td>0.65</td>
<td>0.23</td>
</tr>
<tr>
<td>15</td>
<td>0.42</td>
<td>0.27</td>
</tr>
<tr>
<td>16</td>
<td>0.58</td>
<td>0.23</td>
</tr>
<tr>
<td>17</td>
<td>0.23</td>
<td>0.31</td>
</tr>
<tr>
<td>18</td>
<td>0.92</td>
<td>0.27</td>
</tr>
<tr>
<td>19</td>
<td>0.77</td>
<td>0.35</td>
</tr>
<tr>
<td>20</td>
<td>0.35</td>
<td>0.50</td>
</tr>
<tr>
<td>21</td>
<td>0.27</td>
<td>0.54</td>
</tr>
<tr>
<td>22</td>
<td>0.19</td>
<td>0.27</td>
</tr>
<tr>
<td>23</td>
<td>0.38</td>
<td>0.27</td>
</tr>
<tr>
<td>24</td>
<td>0.23</td>
<td>0.50</td>
</tr>
<tr>
<td>25</td>
<td>0.15</td>
<td>0.38</td>
</tr>
<tr>
<td>26</td>
<td>0.23</td>
<td>0.42</td>
</tr>
<tr>
<td>27</td>
<td>0.54</td>
<td>0.27</td>
</tr>
</tbody>
</table>

Fig. 9 – Graphical representation of sub-sector significance.
7. Conclusions

A critical infrastructure is a complex system of socially significant elements, sub-sectors, sectors and their relationships, all of which are essential to ensuring national security, a thriving economy and the continued provision of basic societal needs. Critical infrastructure systems are constantly exposed to the negative effects of incidents and their failures can have significant impacts on society. The impact effects can be one-way (i.e., simple impacts) or multi-way (i.e., synergistic effects).

Quantifying the synergistic effects due to critical infrastructure failures is an extremely important component of critical
infrastructure protection research because these effects can have disastrous impacts on society. The SYNEFIA methodology presented in this paper is designed to quantitatively assess the synergistic effects of critical infrastructure failures. The theoretical basis of the methodology is provided by the notion of symmetrical operational impact, where the impact on society is directly proportional to the significance of a sector, sub-sector or element of a critical infrastructure system. The methodology algorithm incorporates five phases: (i) critical infrastructure sub-sector identification; (ii) critical infrastructure sub-sector correlation analysis; (iii) critical infrastructure sub-sector significance determination; (iv) impact evaluation; and (v) synergistic effect determination. This application of the methodology to the critical infrastructure of the Czech Republic demonstrates its intuitive appeal and utility. The quantitative determination of the synergistic effects of disruptions to multiple sub-sectors of the Czech infrastructure is of value to researchers, government officials as well as critical infrastructure owners and operators.

The research on synergy and synergistic effect quantification should stimulate the development of sophisticated impact modeling and simulation techniques and tools. An example is dynamic functional modeling, which is currently used by the critical infrastructure protection community primarily for assessing basic impacts – without any synergies. The injection of the SYNEFIA methodology into current techniques and tools will enhance the accuracy and fidelity of impact assessments as well as the prioritization of critical infrastructure protection, response and resilience efforts.

Acknowledgment

This work was conducted under the Research Project V120152019049, RESILIENCE 2015: Dynamic Resilience Evaluation of Interrelated Critical Infrastructure Subsystems, supported by the Ministry of the Interior of the Czech Republic over the period 2015–2019.

References