Doctoral Dissertation Supplement

Credit Risk Management Based on Copula Functions

Field of study: Finance

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VSB – TECHNICAL UNIVERSITY OF OSTRAVA
FACULTY OF ECONOMICS

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1 Introduction

The rapid development of the financial globalization has made financial market around the world more liberate and open than ever and further on integrated into one as a whole, which deepens their interdependence. Capital flow becomes freer and quicker in the world, facilitating the scale and the efficiency of the financial market. However, the degree of opening and the operation mechanism of the financial market vary from different countries. The volatile of one financial market can easily and fast influence the whole financial market due to the financial contagion effect. Therefore, the financial risks faced by the financial institutions become more diversified and complicated.

Financial risks usually include credit risk, market risk, liquidity risk, and operational risk. Credit risk is the potential loss for a financial institution when the borrower fails to meet the obligations in accordance with agreed terms. It is the oldest and largest risk that most financial institutions are faced with. With the increasing attention to the significance of credit risk, there have been numerous studies in measuring and managing credit risk. Many researchers have been developed a series of effective quantitative models for credit risk. Nowadays, researchers are still focusing on the improvement of credit risk models in order to better quantify and then better manage credit risk.

Credit risk is diversified and complicated, displaying nonlinear, asymmetric, time-varying and tail dependence modes. Therefore, the simple linear correlation based on the traditional assumption of the normal distribution is not sufficient and accurate enough to measure credit risk. Copula function is a useful tool to describe dependence structure and determine the higher order joint default probabilities. The propose of the copula theory can be traced back to 1959 by Sklar to link the multivariate distribution and copula function according to the marginally uniform representation. One of the most influential papers on copulas in Finance is written by Embrechts, McNeil and Straumann (2002). Since then, there are an enormous number of papers studying the copula theory in Finance and Economics. Joe (1997) and Nelsen (2006) provide clear and detailed introductions to copulas and their mathematical and statistical elements, and Cherubini et al. (2004) applies copulas to mathematical finance and financial derivatives pricing.

Since that the fat tails and excess kurtosis in the distribution of a single random variable usually increases the probability of extreme events, the non-zero tail dependence of a portfolio also increases the probability of joint extreme events. Cherubini and Luciano (2001), Embrechts et al. (2003), and Embrechts and Höing (2006) study the Value-at-Risk (VaR) of portfolios based on copula functions. Moreover, Rosenberg and Schuermann (2006) uses the method of copulas to construct the joint risk distribution in integrated risk management for financial institutions by aggregating risk types, including market, credit, and operational risk. McNeil et al. (2015) provides a readable textbook of copulas and risk management.
There is a specialized financial market for the credit derivatives called the derivatives market. A typical credit derivative is the credit default swap (CDS) and a portfolio credit derivative is the collateralized debt obligation (CDO). The primary function of credit derivatives is the securitization of credit risk, which means transforming credit risk into securities bought or sold by investors. The derivatives market reached a peak in the period leading up to the 2007-2009 global credit crisis. Although financial activities on CDOs has slowed down after the crisis, CDOs and related financial products still remain significant roles in credit risk management. A recent textbook written by Cherubini et al. (2004) is the first book to address the mathematics of copula functions illustrated with financial applications, such as the derivative pricing and the credit risk analysis. CDO is typically priced under the framework of reduced form models associated with copula functions. The one factor Gaussian copula, proposed by Li (2000), has been proved a benchmark for CDO pricing in a semi-analytical approach. Other variations on factor copulas can be found in Andersen and Sidenius (2004), considering certain nonlinear factor structures, and in Rogge and Schönbucher (2003) and Laurent and Gregory (2005), presenting factor copulas for modeling times-to-default. Applications of copulas in other derivatives markets include Rosenberg (2003), Bennett and Kennedy (2004), van den Goorbergh et al. (2005), Salmon and Schleicher (2006), Grégoire et al. (2008) and Taylor and Wang (2010).

Consider the fact that the conditional volatility of financial time series changes through time, it is necessary to develop the models that also allow the conditional copula to change through time, which is so-called the time-varying copula model. The autoregressive conditional heteroskedasticity (ARCH) model is proposed by Engle (1982) to describe the conditional second moment of the financial time series to reflect the time-varying and clustering fluctuation. Time-varying copula models are considered in Patton (2001, 2004 and 2006), Jondeau and Rockinger (2006), Ausin and Lopes (2010), Christoffersen et al. (2012) and Creal et al (2013). Manner and Segers (2011) and Hafner and Manner (2012) introduce the stochastic dynamic models, which are analogous to the stochastic volatility models in Shephard (2005). Manner and Reznikova (2012) brings together different specifications for copula models with time-varying dependence structure and compares the applicability of each particular model in different cases.
2 Objective and Structure

The general objective of the doctoral thesis is to investigate the credit risk measurements combined with the copula functions. The first subgoal is to compare the original credit risk model and the credit risk model based on copulas; the second subgoal is to compare the ability of pricing of credit derivatives under copula framework; and the third subgoal is to extend the constant copulas to the dynamic copulas and verify the best copula for given portfolios.

The thesis consists of four chapters that concern topics on credit risk management based on copula functions. Chapter 2 basically describes the copula theory. Chapter 2.1 and 2.2 present two fundamental types copulas, namely elliptical copulas and Archimedean copulas, theoretically and graphically. Chapter 2.3 discusses how to fit a copula to data. There are three main methods to calibrate the copula parameters, including maximum likelihood estimator (MLE), Inference functions for margins (IFM), and canonical maximum likelihood (CML). Dependence concepts and measures are considered in Chapter 2.4. Linear correlation is the standard measure for describing the dependence between financial assets regardless of many limitations of normality. Two further classes of measure, namely rank correlations and tail dependence, are directly related to copulas. Rank correlations can be used to calibrate copulas to data, while coefficients of tail dependence aim at measuring the dependence between the joint extreme values.

Chapter 3 is concerned with credit portfolios with a view of credit risk management issues and copula functions. The main theme of portfolio credit risk is modelling the dependence structure of the defaults in the portfolio. We start from the general classification of credit risk models in Chapter 3.1, namely the structural and reduced-form models. The structural models assume that defaults occur when the value of the firm falls below a certain default point and a certain recovery is paid, while the reduced-form models assume that defaults occur exogenously and are unpredictable, and a separately specified recovery is paid. Chapter 3.2 detailedly describes the CreditMetrics™ model proposed by J. P. Morgan in 1997, including risk management framework, credit quality correlation, and Monte Carlo simulation. In Chapter 3.3, the empirical study, we conduct two different portfolios of bonds traded on Frankfurt Stock Exchange (FSE), one is a high-quality portfolio with ten good-rating bonds, and another is a credit-risky portfolio with ten risky bonds. The VaR of two portfolios are calculated and compared both by the original CreditMetrics™ model and by the CreditMetrics™ model based on the copula functions at different confidence levels. We find the empirical evidence that copula functions improve the accuracy of quantifying the VaR and therefore measuring credit risk for the credit-risky portfolio especially when the confidence level is low.
Chapter 4 focuses on CDO pricing under copula framework. Copula models are specifically developed to model portfolio credit derivatives and the Gaussian copula is the most popular copula model and has become a market standard for pricing CDO tranches. In Chapter 4.1, we describe the structure and properties of a CDO. Chapter 4.2 and 4.3 discuss CDO pricing and model calibration in various factor copula models. The large homogeneous portfolio approximation (LHP) is introduced in Chapter 4.4. Chapter 4.5 is the empirical study. The first study is about how the tranche spreads of a CDO will change according to different correlations and recovery rates based on the multinomial Gaussian copula. The second study fits different copula models to price the CDO through the market quotes, including Gaussian copula, double t copula, NIG copula, Gaussian copula with stochastic correlation, Gaussian copula with random factor loadings (RFL), and Clayton copula. The abilities of different factor copula models to fit the market quotes and correlation skew are analyzed and compared in terms of the minimal absolute error sum. The three extensions of the Gaussian copula models outperform and the NIG model can provide the lowest absolute error sum.

Chapter 5 considers dynamic copula models to describe the dependence structure of the nonlinear, asymmetric, and dynamic features of financial time series. Chapter 5.1 firstly introduces the multivariate copula-ARCH models and the extensions include copula-GARCH, copula-IGARCH, and copula-EGARCH models. In Chapter 5.2, we describe time-varying copula models to form a dynamic conditional joint distribution. The bivariate time-varying Gaussian copula model and the bivariate time-varying symmetrized Joe-Clayton (SJC) copula model are presented as two typical examples. Chapter 5.3 extends the topic of dynamic copulas to Markov regime switching (MRS)-copula-GARCH model to draw the probabilistic inference in the form of a nonlinear iterative filter. Chapter 5.3 provides three empirical studies of dynamic copula models. The first and second study is the comparison of constant copulas, time-varying copulas, and time-varying copulas with Markov regime switching by ranking their negative log-likelihood. We find that time-varying copulas with Markov regime switching can perform best in most cases, followed by time-varying copulas. The third study analyzes the dependence of constituents of the FTSE 100 Index by the time-varying copula-GARCH models. The performances of selected copula models are evaluated by the negative log-likelihood and a powerful test named the Hit test is applied. The results suggest that the time-varying SJC copula with the minimal negative log-likelihood is the best.
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Chapter 3 starts from the description of credit risk models of a single obligor, namely the structural and reduced-form models. Then portfolio credit risk models come to the fore and the important issue of dependence between default events is concerned. The CreditMetrics™ model, an industry model based on credit ratings, is introduced in detail. In the empirical study, the CreditMetrics™ model is applied to two well diversified portfolios consisting of ten bonds traded on Frankfurt Stock Exchange (FSE) to estimate the VaR of the portfolio at different significance levels. One portfolio is of high quality with ten good-rating bonds, while another is of credit risky with ten risky bonds. After that, CreditMetrics™ is concerned with the copula functions to recalculate the correlation matrix and the real distribution of the portfolio value. The VaR of two different portfolios by both the original CreditMetrics™ model and the CreditMetrics™ model based on the copula functions are compared and analyzed.

Chapter 4 starts from the basic structure of CDO, then we introduce the pricing of CDO under copula framework. Since the key issue for pricing CDO is to determine the cumulative loss distribution function $F_\infty(x)$, the factor copula models, including the additive factor copulas and Archimedean copulas, are discussed. More specifically, there are seven selected factor copula models, namely Gaussian copula, t copula, double t copula, NIG copula, Gaussian copula with stochastic correlation, Gaussian copula with random factor loadings, and Clayton copula. Besides, the large homogeneous portfolio approximation (LHP) by Vasicek (1987) is introduced to derive the approximating distribution of the portfolio loss. There are two main parts in the empirical study. The first part analyzes how the tranche spreads of a CDO will change in consider with different correlations and recovery rates. And then the second part focuses on the ability of different advanced factor copula models to fit the market quotes and correlation skew.

In Chapter 5, we firstly introduce the copula-ARCH model, including copula-GARCH model, copula-IGARCH model, and copula-EGARCH model. Then we turn to the time-varying copula model to form a dynamic conditional joint distribution. Two typical time-varying copula models include the time-varying Gaussian copula model and the time-varying symmetrized Joe-Clayton (SJC) copula model. Besides, an algorithm named Markov regime switching model (MRS) is proposed to model the changes in regime. The combination of MRS and ARCH process is the MRS-ARCH$(q)$ model, which is extended to the MRS-copula-GARCH$(k,p,q)$ later. In the empirical study, we conduct three studies to verify and find the best copula model for the asset portfolio distribution. The first two focus on the comparison of the constant copula, the time-varying copula, and the time-varying copula with Markov regime switching in terms of their negative log-likelihood. The lower the negative log-likelihood, the better performance of the copula model. The third study is about the dependence of constituents of the FTSE 100 Index by the time-varying copula-GARCH model. What's more, in order to test the goodness-of-fit of the copula models, a powerful test named the Hit test is applied.
5 Methods Applied

CreditMetrics™

The CreditMetrics™ model, proposed by the U.S. bank J. P. Morgan in 1997 originally, is a tool for estimating the distribution of changes in the market value of a portfolio of credit exposures based on the data for migration rates, default rates, and spreads of borrowers with various given rating categories.

Given some confidence level \(\alpha \in (0, 1)\), value-at-risk (VaR) of a portfolio with loss \(L\) at the confidence level \(\alpha\) is given by the smallest number \(l\) such that the probability that the loss \(L\) exceeds \(l\) is no larger than \(1 - \alpha\). Formally,

\[
VaR_{\alpha}(L) = \inf\{P(L > l) \leq 1 - \alpha\}. \tag{1}
\]

In general, there are four steps to calculate the credit risk for a portfolio by using CreditMetrics™:

1) Credit rating migration: It is necessary to specify both the default possibility and the possibilities that companies in one category migrate to other no matter how many rating categories or how these categories are constructed.

2) Calculation of the future value of a bond: In the event of upgrades or downgrades, the change in credit spread is estimated based on a straightforward future value bond revaluation. The future value of a bond can be calculated as

\[
FV = C + \frac{C}{(1+r)} + \frac{C}{(1+r)^2} + \cdots + \frac{C+M}{(1+r)^n}, \tag{2}
\]

where \(C\) is coupon payment, \(n\) is number of payments, \(r\) is forward yield, \(M\) is value at maturity or par value, and \(C + M\) is nominal value.

3) Derivation of the yield curves: the \(n\)-year interest rate \(r^F_n\) is

\[
r^F_n = (1 + r^F) \cdot \left\{ \frac{1-RR \sum_{i=1}^{n} \frac{p^j_{i}-p^j_{i-1}}{(1+r^F)^j}}{1-p^F_n} \right\}^{-1} - 1. \tag{3}
\]

4) Credit risk estimation: According to what we have already obtained from previous two steps, it is able to obtain the likelihoods of all possible rating migration, including upgrades, downgrades, and default, and the distribution of value with each rating migration.

The Cholesky decomposition, also named Cholesky factorization, is used in the Monte Carlo simulations. In the case of two variables only, \(A\) and \(B\), the covariance matrix can be decomposed into two triangular matrices \(A\) and \(A^T\) as

\[
\Sigma = \begin{bmatrix} \sigma_A^2 & \sigma_{AB}^2 \\ \sigma_{AB}^2 & \sigma_B^2 \end{bmatrix} = \begin{bmatrix} \sigma_A \sqrt{\sigma_B - \left(\frac{\sigma_{AB}}{\sigma_A}\right)^2} & \sigma_{AB} \sqrt{\sigma_A - \left(\frac{\sigma_{AB}}{\sigma_B}\right)^2} \\ 0 & \sigma_B \sqrt{\sigma_A - \left(\frac{\sigma_{AB}}{\sigma_B}\right)^2} \end{bmatrix} = AA^T. \tag{4}
\]

Similarly, the correlation matrix can be decomposed as
\[
\Sigma = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \rho & (1 - \rho^2)^{\frac{1}{2}} \end{bmatrix} \cdot \begin{bmatrix} 1 & \rho \\ 0 & (1 - \rho^2)^{\frac{1}{2}} \end{bmatrix}.
\]

(5)

The following equations are used to calculate single element in the Cholesky decomposition matrix of a portfolio with \( n \) assets:

\[
p_{ii} = (\sigma_{ii} - \sum_{k=1}^{i-1} p_{ik}^2)^{\frac{1}{2}}, \forall i = 1, 2, \ldots, n,
\]

(6a)

\[
p_{ij} = (\sigma_{ii} - \sum_{k=1}^{i-1} p_{ik} \cdot p_{jk}) \cdot p_{ii}^{-1}, \forall i, j = 1, 2, \ldots, n,
\]

(6b)

\[
p_{ij} = 0, \forall i > j,
\]

(6c)

where \( p_{ii} \) and \( p_{ij} \) are single element of the Cholesky decomposition matrix.

Although the analytical method is accurate, it is only suitable for a small portfolio. Monte Carlo simulation is therefore needed. This method is based on a simulation of a large number of scenarios. The process of simulation for estimating the distribution of portfolio values consists of the following steps:

1) Generate \( N \) uniformly distributed random variables \( \varepsilon_{i1}, \varepsilon_{i2}, \ldots, \varepsilon_{iN} \) ranging from 0 to 1, convert them into normally distributed variables \( \xi_{i1}, \xi_{i2}, \ldots, \xi_{iN} \) by \( \xi_i = \varphi^{-1}(\varepsilon_i) \), and combine them into a vector \( \xi \);

2) Compute the Cholesky decomposition matrix \( \mathbf{A} \) based on the correlation matrix according to equations (5) and (6);

3) Transform the vector \( \xi \) into the vector \( \mathbf{x} = \xi \cdot \mathbf{A} \) containing possible scenarios for the correlated asset value returns;

4) Find the thresholds corresponding to various rating categories based on the probability transition matrix by the inverse of the normal distribution function;

5) Compare each value of the vector \( \mathbf{x} \) with the thresholds to determine the simulated rating category and then simulated future value;

6) Compute the future value of the portfolio in each scenario by summing up simulated future values of \( n \) assets and generate the distribution of future value of the portfolio.

**Pricing of CDO under copula framework**

Consider a CDO of \( n \) underlying assets with default times \( \tau_i \), loss given default \( \delta_i \), and notional amount \( A_i \). The marginal probability distribution functions of the underlying assets at the time of default are \( F_1(t_1), F_2(t_2), \ldots, F_n(t_n) \). The joint distribution function is \( F \). There exists a copula \( C: [0,1]^n \rightarrow [0,1] \) such that:

\[
F(t_1, t_2, \ldots, t_n) = C(F_1(t_1), F_2(t_2), \ldots, F_n(t_n)).
\]

(7)

Generate \( n \) uniformly distributed random variables \( U_i \) based on the selected copula first and then transform \( U_i \) into default times \( \tau_i \) in ascending order. Compute the default intensity \( \lambda(t) \) and default times \( \tau_i \) is given by \( \tau_i = -\frac{\ln U_i}{\lambda} \). The cumulative loss of the portfolio up to time \( t \) is given by:

\[
L(t) = \sum_{i=1}^{n} \delta_i A_i \mathbf{1}_{(\tau_i \leq t)}, \forall i = 1, 2, \ldots, n.
\]

(8)
Assume that there is a tranche \( \gamma \) and the attachment/detachment points are denoted by \((K_L^\gamma, K_u^\gamma)\), then the cumulative loss of the tranche is:

\[
L(K_L^\gamma, K_u^\gamma, t) = \begin{cases} 
0, & \text{if } L(t) < K_L^\gamma \\
L(t) - K_L^\gamma, & \text{if } K_L^\gamma \leq L(t) \leq K_u^\gamma \\
K_u^\gamma - K_L^\gamma, & \text{if } L(t) > K_u^\gamma
\end{cases}
\]

It can be summarized as the payoff a call spread:

\[
L(K_L^\gamma, K_u^\gamma, t) = \max\{\min[L(t), K_u^\gamma] - K_L^\gamma, 0\} = \left\{\min[L(t), K_u^\gamma] - K_L^\gamma\right\}^+. 
\]

The expected value of the cumulative tranche loss with the continuous portfolio loss distribution function \( F_L(x) \) is:

\[
E[L(K_L^\gamma, K_u^\gamma, t)] = \frac{1}{K_u^\gamma - K_L^\gamma} \int_{K_L^\gamma}^{K_u^\gamma} [\min(x, K_u^\gamma) - K_L^\gamma] dF_L(x) \\
= \frac{1}{K_u^\gamma - K_L^\gamma} \left[ \int_{K_L^\gamma}^{1} (x-K_u^\gamma) dF_L(x) - \int_{K_u^\gamma}^{1} (x-K_L^\gamma) dF_L(x) \right] 
\]

The expected value of the default leg and the premium leg can be computed by:

\[
DL = E\left[ \int_0^T B(0, t) dL(K_L^\gamma, K_u^\gamma, t) \right], \\
PL = E\left[ \sum_{i=1}^n s \Delta t; B(0, t) \min\left\{\max[K_u^\gamma - L(t), 0], K_u^\gamma - K_L^\gamma\right\} \right],
\]

where \( T = t_n \) is the maturity, \( B(0, t) \) is the discount factor until time \( t \), \( s \) is the par spread of the tranche, \( \Delta t_i = t_i - t_{i-1} \), and \( L(t_i) = \delta_{iA_i} \mathbf{1}(t_i \leq s t) \).

Based on the general semi-analytic approach, each tranche conducts a premium so that the premium leg equals the default leg, namely \( PL = DL \). The par spread \( s^* \) is therefore:

\[
s^* = \frac{E[\int_0^T B(0, t) dL(K_L^\gamma, K_u^\gamma, t)]}{E[\sum_{i=1}^n s \Delta t; B(0, t) \min\left\{\max[K_u^\gamma - L(t), 0], K_u^\gamma - K_L^\gamma\right\}]}.
\]

It is clear that the key issue for pricing CDO is to determine the cumulative loss distribution function \( F_L(x) \), which is the important element of the expected value of the cumulative tranche loss \( E[L(K_L^\gamma, K_u^\gamma, t)] \). However, it is not easy to derive \( F_L(x) \) because of influences of the default correlation between the reference entities. Thus, the remaining introduce factor copula model to derive the portfolio cumulative loss distribution \( F_L(x) \).

**Factor copula model**

One factor copula model is a copula model with the latent variables decomposed into one systematic or common factor and \( n \) idiosyncratic factors. It assumes that those factors are distributed based on a certain copula function. The value of the \( i \)-th asset is \( V_i = f(Y, Z_i), \forall i = 1, 2, \cdots, n \), where \( Y \) is the systematic factor, \( Z_i \) are the idiosyncratic factors. \( Y \) and \( Z_i \) are mutually independent random variables.

**Additive factor copula** is widely used in CDO pricing. The function \( f(Y, Z_i) \) means the systematic factor \( Y \) and the idiosyncratic factors \( Z_i \) are additive. Therefore, the value of the \( i \)-th asset is:

\[
V_i = \rho_i Y + \sqrt{1 - \rho_i^2} Z_i, \forall i = 1, 2, \cdots, n,
\]
where $\rho_i \in [0,1]$ is the correlation coefficient between the $i$-th asset and the systematic factor. Denote the probability distribution function of $Y, Z_i$, and $V_i$ by $F_Y, F_Z,$ and $F_V$.

Let $K_i$ be the default barrier of the $i$-th asset, the default time is defined as:

$$
\tau_i = \inf\{t \geq 0 : V_i \leq K_i\}, \forall i = 1, 2, \ldots, n,
$$

(16)

The default barrier can be given by $K_i = F_V^{-1}(Q_i(t))$ and $Q_i(t)$ is the $i$-th default probability for the time $t$.

In the general case, the conditional default probability of the $i$-th asset is:

$$
p_i(y) = P\left(Z_i \leq \frac{K_i - \rho_i Y}{\sqrt{1-\rho_i^2}} \bigg| Y = y\right) = F_Z\left[\frac{F_V^{-1}(Q_i(t)) - \rho_i Y}{\sqrt{1-\rho_i^2}}\right].
$$

(17)

**Archimedean copulas** usually have the explicit forms which makes them good at modelling portfolio credit risk. Famous Archimedean copulas include Gumbel, Clayton, and Frank copula. Consider a positive random variable $\bar{Y}$ with probability density $f_\bar{Y}(x)$ and denote the Laplace transform of $f$ by $\psi$. We have $\psi(s) = \int_0^\infty f_\bar{Y}(x)e^{-sx}dx$. Define the value of the $i$-th asset as:

$$
V_i = \psi\left(-\frac{\ln Z_i}{y}\right), \forall i = 1, 2, \ldots, n,
$$

(18)

Then the conditional default probability of the $i$-th asset is:

$$
p_i(y) = P(V_i \leq K_i|Y = y) = \exp\left[-y\psi^{-1}\left[F_V^{-1}(Q_i(t))\right]\right].
$$

(19)

**Loss distribution of the large homogeneous portfolio**

There exists a so-called large homogeneous portfolio approximation (LHP) that allows to derive an analytical situation for the portfolio loss distribution and the then the expected value of the cumulative tranche loss. The approximation, proposed by Vasicek (1987), assumes that the number of the obligors $n$ in the portfolio is extremely large. All obligors are homogeneous, which means they are identical in notional amounts, recovery rates, and unconditional default probabilities. Thus, for additive factor copulas and Archimedean copulas, the approximating distributions of the portfolio loss are:

$$
F_\infty(x) = P[p_i(y) \leq x] = P\left(F_Z\left[\frac{F_V^{-1}(Q_i(t)) - \rho_i Y}{\sqrt{1-\rho_i^2}}\right] \leq x\right) = P\left(y \geq \frac{F_V^{-1}(Q_i(t)) - \rho_i \overline{F_Z^2}(x)}{\rho_i}\right)
$$

(20)

$$
= 1 - F_Y\left[\frac{F_V^{-1}(Q_i(t)) - \rho_i \overline{F_Z^2}(x)}{\rho_i}\right].
$$

$$
F_\infty(x) = P[p_i(y) \leq x] = P[\exp\left(-y\psi^{-1}\left[F_V^{-1}(Q_i(t))\right]\right) \leq x] = P\left[y \geq -\frac{\ln(x)}{\psi^{-1}\left[F_V^{-1}(Q_i(t))\right]}\right] = 1 - F_Y\left[-\frac{\ln(x)}{\psi^{-1}\left[F_V^{-1}(Q_i(t))\right]}\right].
$$

(21)
Copula-GARCH model

Based on the theories of copula and GARCH (generalized ARCH), copula-ARCH model can be extended to copula-GARCH model. Nelson (1990) derives the condition for strict stationarity of GARCH models in the case of the GARCH(1,1) model and Bougerol and Picard (1992) generalizes this to GARCH(p,q). Assume N random sequences \{y_{1t}\}_{t=1}^{T}, \{y_{2t}\}_{t=1}^{T}, \cdots, \{y_{Nt}\}_{t=1}^{T}, the multivariate copula-GARCH(p,q) is

\[ y_{nt} = \mu_{nt} + \varepsilon_{nt}, \quad n = 1, 2, \cdots, N, \quad t = 1, 2, \cdots, T, \]

\[ \varepsilon_{nt} = h_{nt}^{1/2} \xi_{nt}, \]

\[ h_{nt} = \omega_{n} + \sum_{i=1}^{q_n} \alpha_{ni} \varepsilon_{n,t-i}^{2} + \sum_{i=1}^{p_n} \beta_{ni} h_{n,t-i}, \]

\[ (\xi_{1t}, \xi_{2t}, \cdots, \xi_{Nt}) \sim C(t(\xi_{1t}), F_{2}(\xi_{2t}), \cdots, F_{N}(\xi_{Nt})), \]

where \( \xi_{nt} \sim N(0,1). \)

Bivariate time-varying Gaussian copula model

Recall that bivariate Gaussian copula is

\[ C(u,v; \rho) = \int_{-\infty}^{u} \int_{-\infty}^{v} \frac{1}{2\pi\sqrt{1-\rho^2}} \exp \left[-\frac{s^2-2\rho st+t^2}{2(1-\rho^2)}\right] dsdt, \]

where \( \varphi^{-1} \) is the inverse of the standard normal cumulative density function and \( \rho \in (-1,1) \) is the correlation coefficient. \( \rho \) can be either the constant or time-varying.

Patton (2006) proposes that the upper and lower tail dependence parameters each follow something akin to a restricted ARMA(1,10) process:

\[ \rho_{t} = \tilde{\Lambda}(\omega_{\rho} + \beta_{\rho} \cdot \rho_{t-1} + \alpha_{\rho} \cdot \frac{1}{10} \sum_{i=1}^{10} \Phi^{-1}(u_{t-i}) \cdot \Phi^{-1}(v_{t-i})), \]

where \( \tilde{\Lambda} = (1-e^{-x})(1+e^{-x})^{-1} = \tanh(x/2) \) is the modified logistic transformation designed to maintain \( \rho_{t} \in (-1,1) \) and \{\( u_{t}\}_{t=1}^{T}, \{v_{t}\}_{t=1}^{T} \) are the sequences based on probability integral transformation of the observed sequences. \( \rho_{t-1} \) is the regressor to capture any persistence in the dependence parameter, and the mean of the product of the last 10 observations of the transformed variables \( \Phi^{-1}(u_{t-i}) \) and \( \Phi^{-1}(v_{t-i}) \) to capture the variation in dependence. Specifically speaking, if the product of \( \Phi^{-1}(u_{t-i}) \) and \( \Phi^{-1}(v_{t-i}) \) is positive, the point \( (\Phi^{-1}(u_{t-i}), \Phi^{-1}(v_{t-i})) \) lies in the first or third quadrant; if the product of \( \Phi^{-1}(u_{t-i}) \) and \( \Phi^{-1}(v_{t-i}) \) is negative, the point \( (\Phi^{-1}(u_{t-i}), \Phi^{-1}(v_{t-i})) \) lies in the second or fourth quadrant.

Bivariate time-varying SJC copula model

The Joe-Clayton copula, referred to the BB7 copula, is constructed by taking a particular Laplace transformation of Clayton copula. More details can be found in Joe (1997). The distribution function of the bivariate Joe-Clayton copula is

\[ C_{JC}(u,v; \tau^{U}, \tau^{L}) = 1 - \left( \left\{ 1 - (1 - u)^{\kappa} \right\}^{-\gamma} + \left\{ 1 - (1 - v)^{\kappa} \right\}^{-\gamma} - 1 \right)^{-\frac{1}{\gamma}}, \]

where \( \kappa = 1/\log_{2}(2 - \tau^{U}) \) and \( \gamma = -1\log_{2}(\tau^{L}) \). Two parameters, \( \tau^{U} \in (0,1) \) and \( \tau^{L} \in (0,1) \), are dependence measures known as tail dependence. The Joe-Clayton copula allows upper and lower tail dependence to range anywhere from zero to one.
Although the two tail dependence measures are equal, there is still some slight asymmetry in the Joe-Clayton copula because of its functional form. Therefore, Patton (2001) proposes the symmetrized Joe-Clayton (SJC) copula, which is symmetric when \( \tau^U = \tau^L \):

\[
C_{SJC}(u, v; \tau^U, \tau^L) = 0.5 \cdot [C_{JC}(u, v; \tau^U, \tau^L) + C_{JC}(1 - u, 1 - v; \tau^U, \tau^L)] + u + v - 1.
\] (26)

**Markov regime switching-copula-GARCH model**

Rodriguez (2007) models nonlinearity and asymptotic dependence by use of copulas combined with Markov switching parameters to study financial contagion. The model can be denoted by MRS-copula-GARCH\((k,p,q)\) for simplification. Generally, MRS-copula-GARCH\((2,1,1)\) is effective enough to describe the marginal distribution of the financial sequences. Consider two financial sequences \( \{y_{nt}\}_{t=1}^T, n = 1,2, \) the bivariate MRS-copula-GARCH\((2,1,1)\) is

\[
y_{nt} = a_n + y_{n,t-1} + u_{nt}, \quad t = 1,2,\cdots T,
\]

\[
u_{nt} = \sqrt{g_{nt}}\epsilon_{nt},
\]

\[
e_{nt} = \sqrt{h_{nt}}\xi_{nt}, \quad \xi_{nt} \sim i.i. N(0,1),
\]

\[
h_{nt} = \omega_n + \alpha_n \epsilon_{n,t-1}^2 + \beta_n h_{n,t-1}.
\] (27a-d)

Assume that the dependence between the financial sequence \( \{y_{1t}\}_{t=1}^T \) and \( \{y_{2t}\}_{t=1}^T \) is different under different switching regimes, then the copula functions in the bivariate MRS-copula-GARCH\((2,1,1)\) is

\[(\xi_{1t}, \xi_{2t}) \sim C_t(\phi(\xi_{1t}), \phi(\xi_{2t}); \kappa(s_t)),
\] (28)

where \( C_t(\cdot, \cdot) \) is a bivariate copula function and \( \kappa(s_t) \) is the parameter vector at the unobserved state.
6 Summary of Results and Conclusion

Chapter 3 aims at combining the CreditMetrics™ model and the copula functions to better estimate credit risk. We firstly start from the basic description of credit risk models of a single obligor, including the structural and reduced-form models. Then the framework of the well-known CreditMetrics™ model is described in detail, namely credit rating migration, calculation of the present value of a bond, calculation of the discount rate, and credit risk estimation. Credit quality correlation and Monte Carlo simulation are discussed as well.

We employ the CreditMetrics™ model to two different portfolios, one is a high-quality portfolio with ten good-rating bonds, and another is a credit-risky portfolio with ten risky bonds. Bonds in both portfolios are traded in Frankfurt Stock Exchange and the time horizon is from 9th of October, 2017 to 8th of October, 2018. The total nominal value is fixed at 10 million euro and each bond is represented in 1 million euro equally. After that, the CreditMetrics™ model is concerned with the copula functions to recalculate the correlation matrix and most of correlation coefficients are higher than those in the original correlation matrix. Besides, we try to remove the implicit assumption of the normal distribution in the original CreditMetrics™ model and find the real distribution of the portfolio value to conduct the Monte Carlo simulation using the same uniformly distributed random variables. The real distribution is still the normal distribution for the high-quality portfolio, while it results in the Beta distribution for the credit-risky portfolio. The VaR of the high-quality portfolio are logically lower than the VaR of the credit-risky portfolio no matter at which confidence level, because the riskier the portfolio, the greater value of the VaR. Besides, the original VaR of both portfolios are lower than the VaR calculated based on the copula functions at different confidence levels, which illustrates that the credit risk is underestimated in the original CreditMetrics™ model for both portfolios. Moreover, the difference between the original VaR and the VaR based on the copula functions is more obvious for the credit-risky portfolio, especially when the confidence level is 95%. In my experiment, therefore, it is reasonable to combine the original CreditMetrics™ model and the copula functions for the credit-risky portfolio at any confidence levels, especially when the desired confidence level is 95%. We can also expect that results should probably be similar for more experiments.

Chapter 4 starts from the basic structure of CDO and the pricing of CDO under copula framework. During the derivation of the CDO spread, it is not difficult to find that the core for pricing CDO is to determine the cumulative loss distribution function \( F_{\infty}(x) \), which leads to the development of the factor copula models. There are two main types of the factor copula models, namely the additive factor copulas and Archimedean copulas. Seven factor copula models are described in detail, including Gaussian copula, \( t \) copula, double \( t \) copula, NIG copula, Gaussian copula with stochastic correlation, Gaussian copula with random factor loadings, and Clayton copula. In order to derive an analytical situation for the approximating distribution of the portfolio loss, the large homogeneous portfolio approximation (LHP) is introduced as well.
We conduct two analyses, one is the analysis of how the tranche spreads of a CDO will change with different correlations and recovery rates, another is the ability of selected advanced factor copula models to fit the market quotes and correlation skew. In the first analysis, both the correlations and recovery rates are given from 0 to 0.8 with the increment of 0.2, and the default time is generated by the multinomial Gaussian copula with varying correlation matrices. There are only three tranches for simplification. The tranche spreads for each tranche are summarized in a matrix with 5 rows and 5 columns. The relationships between CDO spreads and correlations are different for different tranches. The CDO spreads increase with an increase in correlation and a decrease in recovery rate for the senior tranche, while the converse is true for the equity tranche. For the mezzanine tranche, the performance is similar to the equity tranche in the relationship between the spreads and correlations, while the performance is similar to the senior tranche in the relationship between the spreads and recovery rates. These results can be explained by the shape of the loss distribution, which is an increasing function of the correlation and a decreasing function of the recovery rate under certain assumptions.

The second analysis aims to compare the ability of different advanced factor copula models to fit the market quotes and correlation skew. The selected copula models include Gaussian copula, double $t$ copula, NIG copula, Gaussian copula with stochastic correlation, Gaussian copula with random factor loadings, and Clayton copula. The Dow Jones iTraxx Europe tranches with 5-year maturity from 5th of January 2015 to 10th of May 2016 is used as the input data. The recovery rate and the risk-free interest rate are uniformly set at 40% and 5% respectively. Six copula models, based on the LHP assumption, are analyzed and compared according to the absolute error obtained by the difference between the model implied spreads and the market spreads. The results of the Gaussian copula and the Clayton copula are quite similar for any tranche, and these two copula models underperform other four copula models. The trend of the absolute error of the double $t$ copula looks like that of both the Gaussian copula and the Clayton copula. The remaining three extensions of the Gaussian copula provide relatively better results. More specifically, the absolute error sum of NIG copula, Gaussian copula with stochastic correlation, and Gaussian copula with random factor loadings are lower and more stable. Besides, the lowest absolute error sum is 4.1 bps given by the NIG model.

Chapter 5 focuses on the dynamic copula models to describe the nonlinear, asymmetric, and dynamic dependence structure in financial markets. The first is the multivariate copula-ARCH model to describe the conditional variance of financial time series. The extensions include copula-GARCH model, copula-IGARCH model, and copula-EGARCH model. Time-varying copula models are usually used to form a dynamic conditional joint distribution, such as the time-varying Gaussian copula model and time-varying SJC copula model. The last is the introduction of the Markov regime switching model (MRS) to draw the probabilistic inference by a nonlinear iterative filter. The combination of MRS, ARCH, and copulas is so-called the MRS-copula-GARCH($k,p,q$) model.
The empirical study consists of three parts. The first study is the comparison of the constant or static copula models and the time-varying copula models. Two stocks issued by ExxonMobil and IBM in FSE are considered from 11th of May 2009 to 15th of March 2019. There are eight constant copulas and three time-varying copulas in the comparison, including Gaussian copula, $t$ copula, Gumbel copula, rotated Gumbel copula, Clayton copula, rotated Clayton copula, Frank copula, SJC copula, time-varying Gaussian copula, time-varying rotated Gumbel copula, and time-varying SJC copula. These eleven copulas are ranked according to their negative log-likelihood and the optimal copula is the one with the lowest negative log-likelihood. The best one is the time-varying SJC copula. Besides, the result of ranking is identical when these copulas are ranked by AIC and BIC. In most cases, the time-varying copulas outperform the constant copulas.

In the second study, the time-varying copulas with Markov regime switching are applied to the same input data as in the first study for the convenient for the comparison. The MRS is assumed to be two regimes and three new added copulas include time-varying Markov Gaussian, rotated Gumbel, and SJC copula. The fourteen copulas in total can be ranked together based on the negative log-likelihood and the best one is the time-varying rotated Gumbel Markov copula with the lowest negative log-likelihood of -81.0947. The results suggest that the time-varying copulas with Markov regime switching can generally perform even better than the time-varying copulas. In reality, due to the fact that it is more complicated to model a time-varying copula with MRS than a time-varying copula or a constant copula, it is necessary to choose the appropriate copula model considering the specific input data and objectives.

As for the third study, we focus on the dependence structure of the FTSE 100 Index by the time-varying copula-GARCH model. Four components of the FTSE 100 Index are chosen from 15th of June 2012 to 26th of July 2019, including EXPN.L (Experian plc), RMV.L (Rightmove plc), CNA.L (Centrica plc), and CCH.L (Coca-Cola HBC AG). A bivariate time-varying copula-GARCH(1,1)-$t$ model is applied to fit the data since the conditional distribution of financial time series is usually time-varying, volatility clustering, leptokurtosis, and fat-tailed. The Kolmogorov-Smirnov test shows that the GARCH(1,1)-$t$ model is able to well fit the conditional marginal distribution. Then we select the time-varying Clayton copula, the static SJC copula, and the time-varying SJC copula as the desired copula models. The results states that the correlation coefficient $\rho_t$ at time $t$ is closely correlated with the correlation coefficient $\rho_{t-1}$ at time $t - 1$ because the parameter $\beta_\rho$ is high. Besides, the time-varying SJC copula model outperforms other two copula models because of its minimal negative log-likelihood. Furthermore, a powerful test named the Hit test is conducted to test the goodness-of-fit of the copula models. All three copula models pass the test easily, which means they are well-specified, and the time-varying Clayton copula and the time-varying SJC copula perform slightly better than the static SJC copula.
7 List of References


8 List of Author’s Publications and Research


9 Summary

Key words: Credit risk, copulas, factor models, dependence, time series

Credit risk is omnipresent in financial markets, so it is important to accurately measure and manage credit risk. Since the dependence structure of financial time series is usually nonlinear, asymmetric, and time-varying, copula functions can well describe the dependence structure and tail dependence. The good properties of copula functions make them widely used in the dependence analysis and multivariate distribution of portfolio credit risk.

The thesis focuses on the application of copula functions in credit risk measurement and management. The main research works in the thesis can be summarized as follows:

1. The research status of the application of copula functions in Finance were systematically reviewed and the basic properties and theories of copula functions were comprehensively introduced both mathematically and graphically.

2. We described credit risk models briefly, including the structural and reduced-form models, and the CreditMetrics™ model in detail. Two different portfolios consisting of ten bonds traded on FSE were created, one was a good-quality portfolio, and another was a credit-risky portfolio. As summarized in subchapter 3.4, we calibrated the parameters of copula functions to fit the sample data and obtained the correlation matrix based on copulas. Besides, the implicit assumption of normal distribution was removed and Monte Carlo simulation was conducted based on the real distribution of the portfolio value. According to the comparison of the VaR by the original CreditMetrics™ model and the CreditMetrics™ model based on copulas of two portfolios, we found that copula functions helped improve the accuracy of measuring credit risk for the credit-risky portfolio at any confidence levels.

3. We introduced the structure and properties of a CDO and derived the CDO pricing formula in the semi-analytic approach under copula framework. The conditional probabilities of default and the approximating distributions of the portfolio loss associated with selected factor copula models were summarized. Furthermore, we figured out that the CDO spreads increased with an increase in correlation and with a decrease in recovery rate for the senior tranche, while the converse was true for the equity tranche. As for the mezzanine tranche, it was similar to the equity tranche in the relationship between the spreads and correlations, while it was similar to the senior tranche in the relationship between the spreads and recovery rates. After comparing the ability of selected factor copula models to fit the market quotes and correlation skew, we found that three extensions of the Gaussian copula model outperformed others and the lowest absolute error sum was provided by the NIG model.

4. Dynamic copula models were presented, including multivariate copula-ARCH model, time-varying copula model, and MRS-copula-GARCH model. Firstly, we compared the negative log-likelihood of static copulas, time-varying copulas, and MRS-copula-GARCH copulas, and found that MRS-copula-GARCH copulas usually provided the lower negative log-likelihood. Moreover, we used the time-varying copula-GARCH models to study the dependence of constituents of the FTSE 100 Index. We found that the time-varying SJC copula was the best, which provided the lowest negative log-likelihood.
Apart from the achievements of the thesis mentioned above, some questions for further research are put forward as follows:

1. The thesis only applies bivariate copula functions to the measurement of credit risk and fails to study the multivariate copula functions in the reality. It is the research emphasis in the future.

2. In the application of the CreditMetrics™ model, the measurement of credit risk is easily affected by the rating system of the rating agency although the introduction of copula functions is effective. Therefore, it is necessary to study how to measure portfolio credit risk more accurately considering more information about the specific financial situations of bond issuers in the portfolio.

3. It is challenging but meaningful to develop dynamic copulas for modeling high dimensional data in further research.
Summary (in Czech)

Key words: Úvěrové riziko, kopule, faktorové modely, závislost, časové řady

Na finančních trzích je úvěrové riziko všudypřítomné, proto je důležité přesně měřit a řídit úvěrové riziko. Protože struktura závislosti finančních časových řad je obvykle nelineární, asymetrická a časově proměnná, funkce kopule mohou dobře popsat strukturu závislosti a závislost na ocasu. Díky dobrým vlastnostem funkci kopule jsou široce využívány při analýze závislosti a vícerozměrné distribuci portfolia úvěrového rizika.

Diplomová práce se zaměřuje na aplikaci kopulačních funkcí při měření a řízení úvěrového rizika. Hlavní výzkumné práce v práci lze shrnout takto:

1. Systematicky byl přezkoumáván výzkumný stav aplikace funkcí kopule ve financích a základní vlastností a teorie funkcí kopule byly kompletně představeny matematicky i graficky.

2. Krátece jsme popsali modely úvěrového rizika, včetně strukturálních modelů a modelů se sníženou formou a modelu CreditMetrics™ podrobně. Byly vytvořeny dvě různá portfolia skládající se z deseti dluhopisů obchodovaných na FSE, jedno bylo portfolio kvalitní a druhé portfolio úvěrové. Jak je shrnuto v podkapitole 3.4, kalibrovali jsme parametry funkci kopule tak, aby odpovídaly datům vzorku, a získali jsme korelační matici na základě Kopule. Kromě toho bylo odstraněno implicitní předpoklad normální distribuce a simulace Monte Carlo byla provedena na základě skutečné distribuce hodnot portfolia. Podle srovnání VaR s původním modelem CreditMetrics™ a modelem CreditMetrics™ založeným na kopulích dvou portfolií jsme zjistili, že funkce kopule pomohly zlepšit přesnost měření úvěrového rizika pro úvěrově rizikové portfolio na jakékoli úrovni spolehlivosti.

3. Představili jsme strukturu a vlastnosti CDO a odvozili jsme cenový vzorec CDO v semi-analytickém přístupu v rámci copula framework. Byly shrnuty podmíněné pravděpodobnosti selhání a přibližné rozdělení ztráty portfolia spojené s vybranými modely faktorových kopula. Dále jsme zjistili, že rozpětí CDO se zvýšilo se zvýšením korelace a se snížením míry návratnosti pro hlavní tranší, zatímco konverze platila pro tranší vlastního kapitálu. Pokud jde o mezaninovou tranší, byla podobná tranší vlastního kapitálu ve vztahu mezi rozpětí a korelací, zatímco ve vztahu mezi rozpětí a mírou návratnosti byla podobná jako hlavní tranší. Po porovnání schopností vybraných modelů faktorových kopula přizpůsobit se tržním cenám a korelačním skokům jsme zjistili, že tři rozšíření gaussovského modelu kopule překonala ostatní a nejnižší absolutní chybovost byla poskytnuta modelem NIG.


Kromě výše zmíněných výsledků práce jsou kladeny některé otázky pro další výzkum:
1. Diplomová práce aplikuje pouze bivariační kopulační funkce na měření úvěrového rizika a nezkoumá multivariační kopulační funkce ve skutečnosti. Je to výzkumný důraz v budoucnosti.

2. Při použití modelu CreditMetrics™ je měření úvěrového rizika snadno ovlivněno ratingovým systémem ratingové agentury, ačkoli je zavedení funkcí kopula efektivní. Je proto nutné studovat, jak přesněji měřit portfoliové úvěrové riziko, a to s ohledem na více informací o specifické finanční situaci emitentů dluhopisů v portfoliu.

3. Je obtížné, ale smysluplné vyvíjet dynamické kopuly pro modelování vysokorozměrných dat v dalším výzkumu.