Abstract: We present a white-light spectral interferometric technique for measuring the absolute spectral optical path difference (OPD) between the beams in a slightly dispersive Michelson interferometer with a thin-film structure as a mirror. We record two spectral interferograms to obtain the spectral interference signal and retrieve from it the spectral phase, which includes the effect of a cube beam splitter and the phase change on reflection from the thin-film structure. Knowing the effective thickness and dispersion of the beam splitter made of BK7 optical glass, we use a simple procedure to determine both the absolute spectral phase difference and OPD. The spectral OPD is measured for a uniform SiO$_2$ thin film on a silicon wafer and is fitted to the theoretical spectral OPD to obtain the thin-film thickness. The theoretical spectral OPD is determined provided that the optical constants of the thin-film structure are known. We measure also the nonlinear-like spectral phase and fit it to the theoretical values in order to obtain the thin-film thickness.

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References and links

1. Introduction

During the past few years there has been considerable interest in the spectral-domain interference phenomena for light beams from both spectrally broadband and white-light sources that give rise to spectral interference fringes, i.e., modulations in the spectra referred to as spectral interferograms or channelled spectra. The spectral interferogram includes background component related to the light source spectrum and useful component of interference fringes related to the spectral phase difference or OPD between the interfering light beams. The information about the OPD to be measured is contained in unwrapped fringe phase and it can be used for measuring distances and displacements [1, 2] and in optical profilometry [3, 4, 5]. Spectral interferometry involves also measurement of the local fringes periodicity in the vicinity of a stationary phase point (zero-order fringe) that appears in the measured spectral interferogram when the group OPD is close to zero. Using a suitable fitting procedure, the refractive index dispersion of optical media can be measured over a broad spectral range [6, 7]. We extended the technique for measuring the group dispersion of thick optical samples [8] or distances in a strongly dispersive Michelson interferometer [9]. Recently, fiber-optic implementation of the interferometer for distance measurements has been presented [10].

The spectral phase retrieval, which can be performed by various techniques like Fourier transform method [11], phase-locked loop method [11], and the Kalman filtering method [12], is crucial for absolute distance measurements in interferometers with thin films [2, 5] characterized by the phase change on reflection that depends on wavelength and layer thickness. It has been shown for thin-film structures with one and two layers that the thicknesses of these layers can be determined with a high accuracy provided that the refractive indices are known [2]. However, some interferometric configurations are affected by a dispersion error as a result of a small asymmetry in the interferometer with a beam splitter cube that is not perfectly cubic [13]. To specify the effect we introduced the effective thickness of the beam splitter which was measured by a spectral interferometric technique [14].

In this paper, we present a white-light spectral interferometric technique for measuring the absolute spectral phase difference or OPD between the beams in a slightly dispersive Michelson interferometer with a thin-film structure as a mirror. Two spectral interferograms were recorded.
to measure the spectral interference signal from which we retrieved the spectral phase including
the effect of a cube beam splitter and the phase change on reflection from the thin-film structure.
Knowing the effective thickness and dispersion of the beam splitter made of BK7 optical glass,
a simple procedure was used to determine both the absolute spectral phase difference and OPD.
The spectral OPD was measured for a uniform SiO₂ thin film on a silicon wafer and was fitted
to the theoretical spectral OPD to obtain the thin-film thickness. The theoretical spectral OPD
was determined provided that the optical constants of the thin-film structure are known. We
measured also the nonlinear-like spectral phase and used a fit to the theoretical values to obtain
the thin-film thickness.

2. Theory

2.1. Spectral interferogram and interference signal

Consider the mutual interference of two beams from a broadband source at the output of a
slightly dispersive Michelson interferometer with a cube beam splitter of the effective thickness
tₑf [14]. We assume that the geometrical path lengths of the light rays in dispersive glass of the
beam splitter are not the same for both interferometer arms so that the beam splitter can be
represented by an ideal beam splitter and a plate of the same dispersion and of the thickness tₑf
(see Fig. 1). Next, let us consider that one of the mirrors of the interferometer is replaced by a
thin-film structure on a substrate, which is characterized by its complex reflection coefficient
\( r(\lambda) = \sqrt{R(\lambda)} \exp[i \delta_r(\lambda)] \), where \( R(\lambda) \) is the wavelength-dependent reflectivity and \( \delta_r(\lambda) \) is
the wavelength-dependent phase change on reflection.

The spectral intensity (interferogram) \( I(\lambda) \) recorded at the output of the interferometer by a
fiber-optic spectrometer of a Gaussian response function can be expressed as [9]

\[
I(\lambda) = I^{(0)}(\lambda) \left\{ 1 + V(\lambda) \exp \left\{ -(\pi^2/2) [\Delta^2(\lambda) \Delta \lambda_R / \lambda^2]^2 \right\} \cos \left\{ (2\pi/\lambda) \Delta(\lambda) \right\} \right\},
\]

where \( I^{(0)}(\lambda) \) is the reference spectral intensity, \( V(\lambda) \) is the wavelength-dependent visibility
term, \( \Delta \lambda_R \) denotes the width of the spectrometer response function and \( \Delta(\lambda) \) is the wavelength-
dependent OPD between two beams in the Michelson interferometer, which is given by

\[
\Delta(\lambda) = 2L + 2n(\lambda) \tau_{ef} - \lambda \delta_r(\lambda)/(2\pi),
\]
where $2L$ is the difference of path lengths between the interfering beams in the air whose dispersion is neglected and $n(\lambda)$ is the wavelength-dependent refractive index of the beam splitter material. The corresponding group OPD $\Delta g(\lambda)$ satisfies over the wavelength range slightly broader than that of the visible spectrum the approximation $\Delta g(\lambda) \approx 2L + 2N(\lambda)t_{ef}$, where $N(\lambda)$ is the wavelength-dependent group refractive index, which is related to the refractive index $n(\lambda)$ via the equation

$$N(\lambda) = n(\lambda) - \lambda \frac{dn(\lambda)}{d\lambda}.$$  

(3)

The interferometer can be considered as slightly dispersive if the effective thickness $t_{ef}$ is smaller than thickness $t_e = 1.36\lambda^2/[\Delta \lambda R N(\lambda)]$ [9]. The group OPD $\Delta g(\lambda)$ can be used to estimate the period $\Lambda(\lambda)$ of the spectral modulation in the recorded spectrum, which is given by the simple relation

$$\Lambda(\lambda) = \lambda^2 / |\Delta g(\lambda)|.$$  

(4)

The reference spectral intensity $I(0)(\lambda)$ can be simply recorded by adjusting the OPD in the interferometer sufficiently large to lower the overall visibility to zero. Then we can evaluate the spectral interference signal $S(\lambda)$ defined as

$$S(\lambda) = I(\lambda)/I(0)(\lambda) - 1.$$  

(5)

Using Eq (1), we obtain

$$S(\lambda) = V(\lambda)\exp\{-\left(\pi^2/2\right)[\Delta g(\lambda)\Delta \lambda R/\lambda^2]^2 \cos\{(2\pi/\lambda)\Delta(\lambda)\}\}.$$  

(6)

If we consider a thin-film structure (a uniform thin film) on a substrate, multiple reflection takes place and the complex reflection coefficient can be expressed [5, 15] with the use of the wavelength-dependent refractive index of the thin-film structure $n_1(\lambda)$, the real refractive index $n_2(\lambda)$ and the extinction coefficient $\kappa_2(\lambda)$ of the substrate. The phase change $\delta_r(\lambda)$ can be represented as the sum of two contributions

$$\delta_r(\lambda) = 2n_1(\lambda)d + \phi_{nl}(\lambda),$$  

(7)

where $d$ is the thickness of the thin-film structure and $\phi_{nl}(\lambda)$ is the wavelength-dependent nonlinear phase function due to the multiple reflection within the thin-film structure [2, 5]. In the case of a thick film, $\delta_r(\lambda) \approx 2n_1(\lambda)d$ and Eqs. (2) and (7) give the OPD which corresponds to two different media in the individual arms of the interferometer [9].

2.2. Signal processing

Consider now that the measured spectral interference signal $S(\lambda)$ is processed by one of the above-mentioned methods and is represented in the form

$$S(\lambda) = a(\lambda)\cos[\Phi(\lambda)],$$  

(8)

where $a(\lambda)$ is the overall visibility function or the envelope function and $\Phi(\lambda)$ is the unwrapped phase function, which is known with the ambiguity of $m2\pi$, where $m$ is an integer. Using Eqs. (6) and (8), the relation for the spectral OPD $\Delta(\lambda)$ between interfering beams

$$\Delta(\lambda) = [\Phi(\lambda)/(2\pi) + m]\lambda$$  

(9)

has to be fulfilled. The knowledge of the unwrapped phase function $\Phi(\lambda)$, the effective thickness $t_{ef}$ of the beam splitter and the dispersion of its refractive index $n(\lambda)$ can be used, as it results from Eqs. (2) and (9), for determining the interference order $m$ and thus the absolute...
spectral phase difference \( \varphi(\lambda) \) or OPD \( \Delta(\lambda) = (\lambda/2\pi)\varphi(\lambda) \). The interference order \( m \) of such a value has to be chosen so that the spectral OPD \( \Delta(\lambda) \) between beams in the interferometer is linearly dependent on the refractive index \( n(\lambda) \) of the beam splitter material. Moreover, knowing the spectral OPD \( \Delta(\lambda) \) we can construct for a chosen mirror position \( L = L_0 \) the so-called nonlinear-like phase function \( \delta(\lambda) \), which satisfies the relation

\[
\delta(\lambda) = (2\pi/\lambda)[2L_0 + 2n(\lambda)t_{ef} - \Delta(\lambda)].
\]

3. Experimental setup

The experimental setup used in the application of spectral-domain white-light interferometry to measure the spectral phase difference or OPD is shown in Fig. 1. It consists of a white-light source: a halogen lamp HL-2000 (Ocean Optics, Inc.) with launching optics, an optical fiber and a collimating lens, a bulk-optic Michelson interferometer with a cube beam splitter made of BK7 optical glass, a thin-film structure on a substrate, metallic mirror connected to a micropositioner, a microscope objective, micropositioners, a read optical fiber, a miniature fiber-optic spectrometer S2000 (Ocean Optics, Inc.), an A/D converter and a personal computer. The effective thickness of the beam splitter is \( t_{ef} = -10.10 \, \mu m \) and it was determined by a spectral interferometric technique [14] utilizing a Michelson interferometer with two identical metallic mirrors. The negative effective thickness of the beam splitter means that the corresponding dispersive plate is in the other arm of the interferometer than is depicted in Fig. 1.

The thin-film structure is represented by a uniform SiO\(_2\) thin film on the silicon wafer, which was prepared using a dry oxidation process described by the so-called Deal-Grove model [16]. Single-crystal silicon wafer (ON Semiconductor, Czech Republic) is characterized by subsequent parameters: diameter \( (100\pm0.5) \, \text{mm} \), orientation \( (111) \), B doped type P, thickness \( (381\pm25) \, \mu m \) and resistivity \( (0.008\pm0.009) \, \Omega \, \text{cm} \). Before the oxidation, the wafer was cut into \( 40\times40 \, \text{mm} \) square, cleaned by standard methods and then annealed in a furnace at 1200 \(^\circ\text{C}\). According to the model, annealing time was selected in order to prepare SiO\(_2\) thin film of a suitable thickness.

The fiber-optic spectrometer S2000 of an asymmetric crossed Czerny-Turner design with the input and output focal lengths of 42 and 68 mm, respectively, has the spectral operation range from 350 to 1000 nm and includes a diffraction grating with 600 lines per millimeter, a 2048-element linear CCD-array detector with a Schott glass long-pass filter, a collection lens and a read optical fiber. The wavelength of the spectrometer is calibrated so that a third-order polynomial relation between pixel number and wavelength is used. The spectrometer resolution is in our case given by the effective width of the light beam from a core of the read optical fiber: we used the read optical fiber of a 50 \( \mu m \) core diameter to which a Gaussian response function of the width \( \Delta \lambda_R = 3 \, \text{nm} \) corresponds [17]. Spectrometer sensitivity is adjusted by the spectrometer integration time of 5 ms.

4. Experimental results and discussion

In order to demonstrate the principle of obtaining the absolute spectral OPD \( \Delta(\lambda) \) and the nonlinear-like phase function \( \delta(\lambda) \), first we show in Fig. 2(a) the reference spectral intensity \( I^{(0)}(\lambda) \) and the spectral interferogram \( I(\lambda) \) recorded for two suitable OPDs adjusted in the interferometer. Figure 2(b) then shows the interference spectral signal \( S(\lambda) \) evaluated using Eq. (5). It can be seen from Fig. 2(b) that the interference signal reflects the wavelength dependences of both the overall visibility and spectral phase. In the overall visibility function or the envelope of the interference signal the apparent effect of the reflectivity \( R(\lambda) \) of the thin-film structure is present. Similarly, the spectral phase is dependent on the phase change on reflection from the thin-film structure. To retrieve the phase function \( \Phi(\lambda) \), which is crucial for our sub-
sequent considerations, the Kalman filtering method was used [12]. The precision of the phase retrieval is demonstrated in Fig. 3(a), in which the interference signal $S(\lambda)$ is compared with a signal $\cos[\Phi(\lambda)]$. Figure 3(a) illustrates very good agreement between both signals.

Knowing both the unwrapped phase function $\Phi(\lambda)$, the effective thickness $t_{\text{ef}}$ of the beam splitter and its refractive index dispersion $n = n(\lambda)$, the interference order $m$ in Eq. (9) and thus the absolute spectral phase difference $\phi(\lambda)$ or OPD $\Delta(\lambda) = (\lambda/2\pi)\phi(\lambda)$ were determined. We utilized the fact that the OPD $\Delta(\lambda)$ is linearly dependent on the refractive index $n(\lambda)$ as is illustrated in Fig. 3(b). The refractive index $n(\lambda)$ of BK7 optical glass is approximated by the semi-empirical Sellmeier expression [18]. The first estimate of the interference order $m$ in Eq. (9) corresponds to the OPD $\Delta(\lambda_0)$, which is close to the group OPD $\Delta_g(\lambda_0)$ given according to Eq.(4) by the period $\Lambda(\lambda_0)$ at one specific wavelength $\lambda_0$.

Next, the measured absolute spectral OPD $\Delta(\lambda)$ was compared with the theoretical one given by Eq. (2) to determine the thickness $d$ of the SiO$_2$ thin film on the silicon wafer. Dispersion in SiO$_2$ thin film was approximated by the dispersion relation of the Sellmeier-like form with one term [19] and the wavelength-dependent refractive index $n_2(\lambda)$ and extinction coefficient $\kappa_2(\lambda)$ of silicon were approximated by dispersion relations from [20].

Figure 4(a) shows the comparison of the experimental results with the theoretical ones obtained by fitting the theoretical OPD $\Delta(\lambda)$ to the measured OPD $\Delta'(\lambda)$ using the Levenberg-Marquardt least-squares algorithm [21]. The method determines the maximum-likelihood estimate of parameters $L$ and $d$ that minimizes the figure-of-merit function $\chi^2$, defined by

$$\chi^2(L,d) = \sum_{i=1}^{N} [\Delta'(\lambda_i) - \Delta(\lambda_i;L,d)]^2 \quad (11)$$

where $\lambda_i$ are wavelengths at which the spectrum was recorded (the fit was performed from 450 to 950 nm). Figure 4(a) demonstrates very good agreement between theory and experiment with the correlation coefficient as high as 0.99810 and values of two parameters, which are approximately $L = 26505.4$ nm and $d = 392.4$ nm. It is also clearly seen from Fig. 4(a) that the discrepancy between the experiment and theory can be mainly attributed to errors in retrieving the spectral phase difference at the edges of the used spectral range.

The thickness $d$ of the thin film was also evaluated from the nonlinear-like phase function $\delta(\lambda)$, which used the estimate of the mirror position $L_0$. We choose $L_0$ in such a way that the dependence of the OPD $\Delta(\lambda)$ on the refractive index $n(\lambda)$ of BK7 optical glass deviated...
minimally from a linear dependence. Figure 3(b) shows the linear function by the red line and the corresponding mirror position was approximately $L_0 = 25937.2$ nm. The deviation of the OPD $\Delta(\lambda)$ from the linear dependence in Fig. 3(b) is due to the term $\lambda \delta(\lambda)/(2\pi)$ and thus due to the effect of the nonlinear-like phase function $\delta(\lambda)$, which was simply determined from Eq. (10). Figure 4(b) shows the comparison of the experimental results with the theoretical ones obtained by fitting the theoretical nonlinear-like phase function $\delta(\lambda)$ to the measured nonlinear-like phase function $\delta^*(\lambda)$ using the Levenberg-Marquardt least-squares algorithm. The method determines the maximum-likelihood estimate of parameters $L$ and $d$ that minimizes the figure-of-merit function $\chi^2$, defined in a similar way as that in Eq. (11). Figure 4(b) demonstrates very good agreement between theory and experiment with the correlation coefficient as high as 0.98798 and values of two parameters, which are approximately $L=26503.0$ nm and $d=390.7$ nm. The thickness was measured with an estimated uncertainty of $\pm 2$ nm and the minimum thickness that can be measured by this technique is as low as 50 nm when the oscillatory behavior of the non-linear phase function is suppressed.

![Fig. 3. (a) Comparison of the measured spectral interference signal with the theoretical one (red curve) that uses the retrieved spectral phase; (b) The absolute OPD as a function of the refractive index of BK7 glass. Red line is linear fit.](image)

![Fig. 4. The absolute OPD as a function of wavelength together with the corresponding fit (red curve); (b) The nonlinear-like phase as a function of wavelength together with the corresponding fit (red curve).](image)

We have performed additional measurements by a technique of null ellipsometry [15]...
and spectral reflectometry [22]. In the first case we obtained approximately the thickness \(d=389.8\) nm and in the second case in the wavelength range from 390 to 1000 nm approximately the thickness \(d=392.6\) nm (with the correlation coefficient as high as 0.99394). It should be stressed here that we can also use a simplified model with the wavelength-independent constants [23] that gives film thickness with a worse fit and lower accuracy. In order to determine the thicknesses of the SiO\(_2\) thin film precisely, one has to know the exact optical constants for both the SiO\(_2\) thin film and Si substrate. They can be obtained by an additional measurement technique, such as a technique of spectral ellipsometry.

5. Conclusions

We used a white-light spectral interferometric technique for measuring the absolute spectral phase difference or OPD between the beams in a slightly dispersive Michelson interferometer with a thin-film structure as a mirror. We recorded two spectral interferograms to obtain the spectral interference signal and retrieved from it the spectral phase, which includes the effect of a cube beam splitter and the phase change on reflection from the thin-film structure. Knowing the effective thickness and dispersion of the beam splitter made of BK7 optical glass, we used a simple procedure to determine both the absolute spectral phase difference and OPD. We measured the spectral OPD for a uniform SiO\(_2\) thin film on a silicon wafer and fitted it to the theoretical spectral OPD to obtain the thin-film thickness. The theoretical OPD was determined provided that the optical constants of the thin-film structure are known. We measured also the nonlinear-like spectral phase and fitted it to the theoretical values to obtain the thin-film thickness.

The results obtained serve as an illustration of the feasibility of a simple and cost-effective measurement technique based on the recording spectral interferograms by using a fiber-optic spectrometer. It allows to determine the absolute spectral phase difference or OPD in the interferometer and the thickness of a thin-film structure provided that the optical constants of the structure are known. This method, which can be extended to a broader spectral range and which has the primary advantage over a technique such as ellipsometry in its normal incidence configuration, allows for measurements of one- or two-dimensional thickness profiles by applying of the microscope interferometer. It should be stressed that there are other potential applications in profilometry and in reflectometry for measuring the nonlinear-like phase function of complex structures.

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